Computer Engineering Department

Inverse Reinforcement Learning

Mohammad Hossein Rohban, Ph.D.

Spring 2025

Slides are adopted from CS 285, UC Berkeley.

Lecture Outline

1. So far: manually design reward function to define a task
2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
3. Apply approximate optimality model from last time, but now learn the reward!

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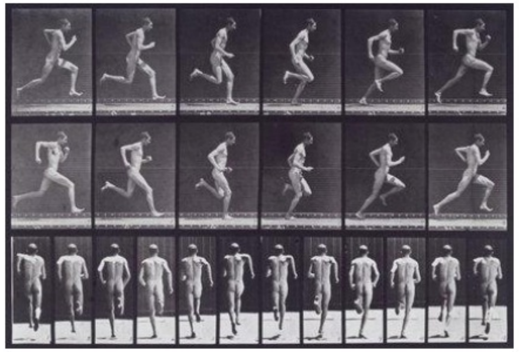
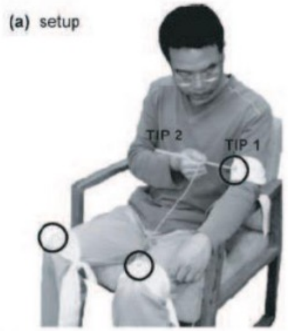
- Goals:
  - Understand the inverse reinforcement learning problem definition
  - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
  - Understand a few practical inverse reinforcement learning algorithms we can use

Modeling Human Behavior with Optimal Control



Muybridge (c. 1870)

Mombaur et al. '09



Li & Todorov '06

Ziebart '08

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

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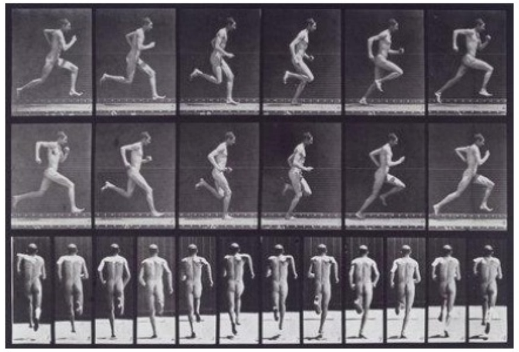
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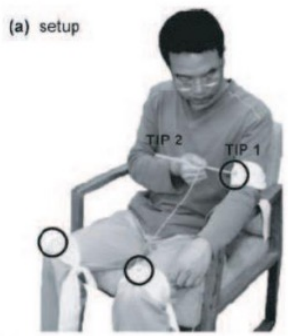

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optimize this to explain the data

$$\pi = \arg \max_{\pi} E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$\mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)$$

Imitation learning vs RL perspective

The imitation learning perspective

Standard imitation learning:

- copy the *actions* performed by the expert
- no reasoning about outcomes of actions

Human imitation learning:

- copy the *intent* of the expert
- might take very different actions!

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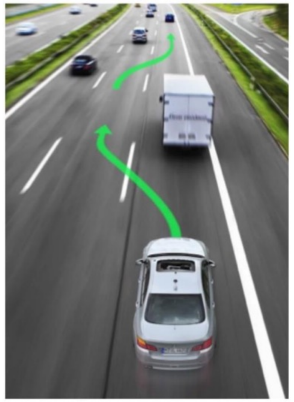
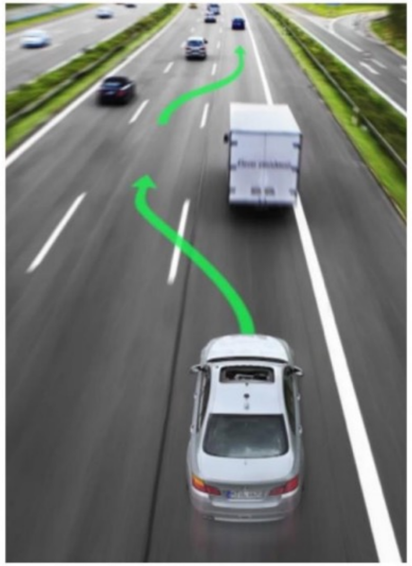


what is the reward?

Inverse Reinforcement Learning

Infer **reward functions** from **demonstrations**



$$\Rightarrow \quad r(\mathbf{s}, \mathbf{a})$$

Inverse Reinforcement Learning

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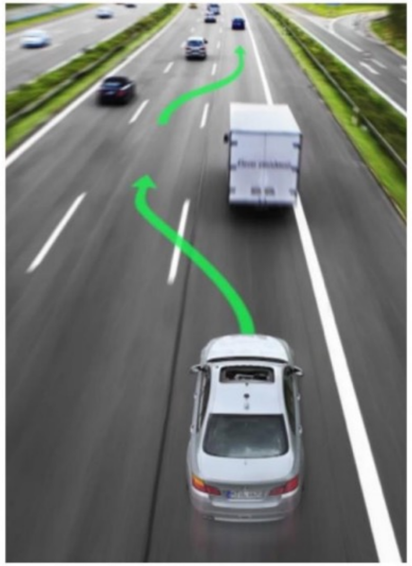
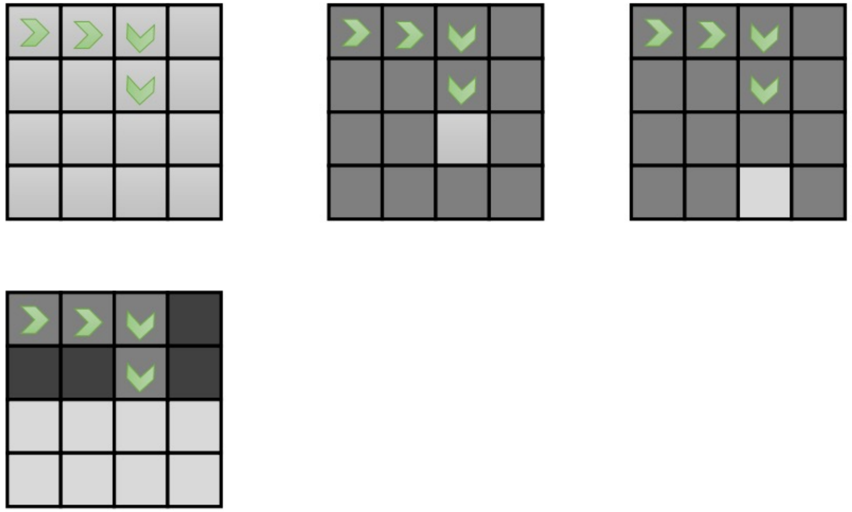


$$r(\mathbf{s}, \mathbf{a})$$

by itself, this is an **underspecified** problem

many reward functions can explain the **same** behavior



Inverse Reinforcement Learning Formulation

"forward" reinforcement learning

inverse reinforcement learning

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"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$

(sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

inverse reinforcement learning

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reward parameters

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linear reward function:

$r_\psi(\mathbf{s}, \mathbf{a}) = \sum_i \psi_i f_i(\mathbf{s}, \mathbf{a}) = \psi^T \mathbf{f}(\mathbf{s}, \mathbf{a})$

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neural net reward function:

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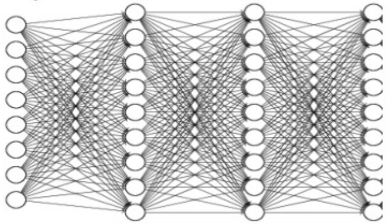

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reward function $r(\mathbf{s}, \mathbf{a})$	samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$
learn $\pi^\star(\mathbf{a}\vert\mathbf{s})$	learn $r_\psi(\mathbf{s}, \mathbf{a})$
	reward parameters
	...and then use it to learn $\pi^\star(\mathbf{a}\vert\mathbf{s})$

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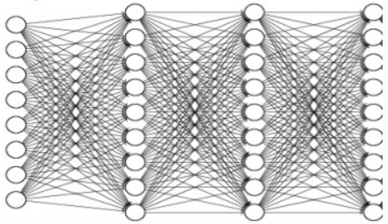

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need to somehow "weight" by similarity between $\pi^\star$ and π

Feature Matching IRL & Maximum Margin

remember the "SVM trick":

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e.g., difference in feature expectations!

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Further reading:

- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

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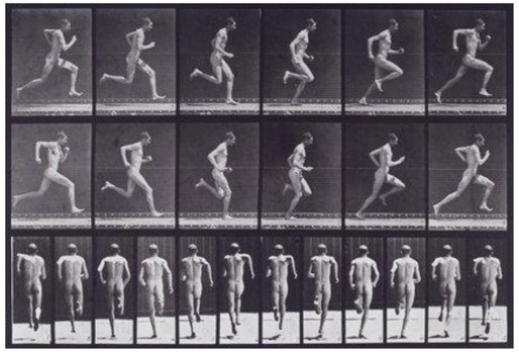
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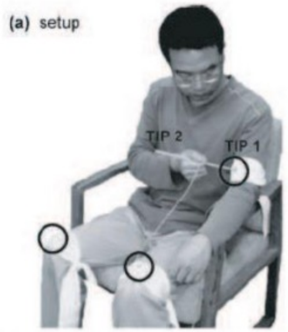

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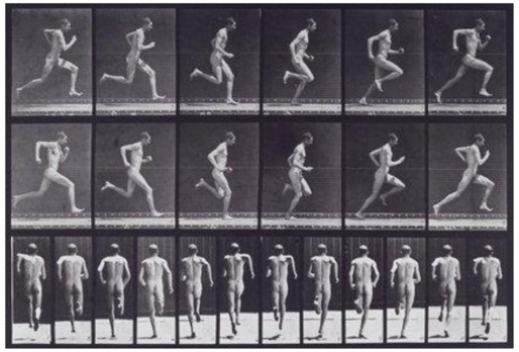

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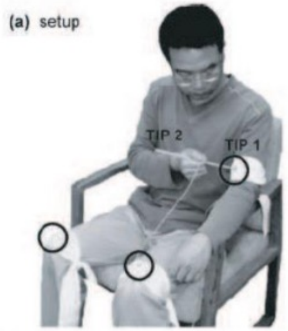

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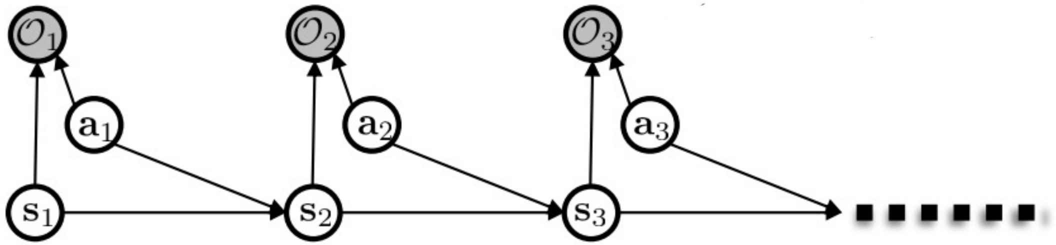

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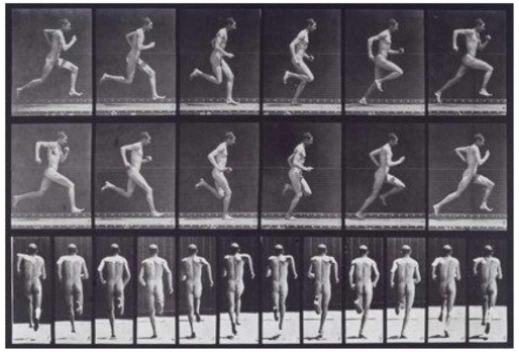

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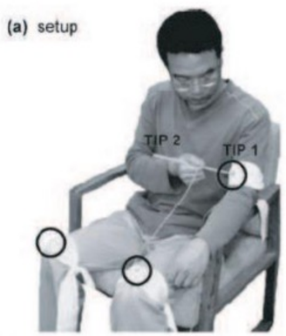

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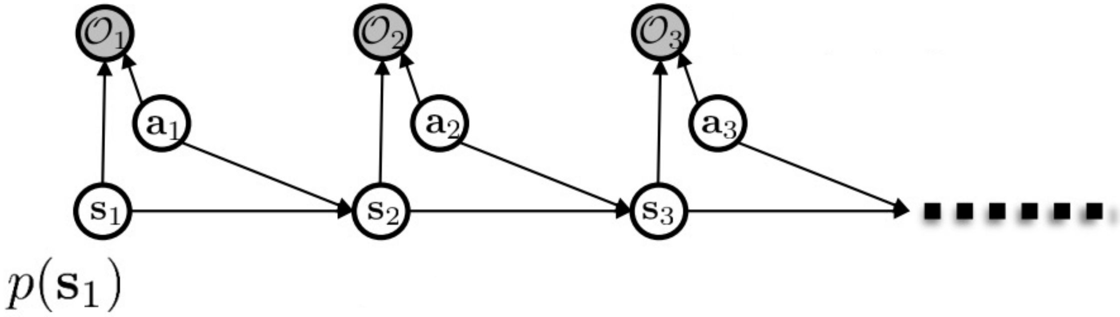
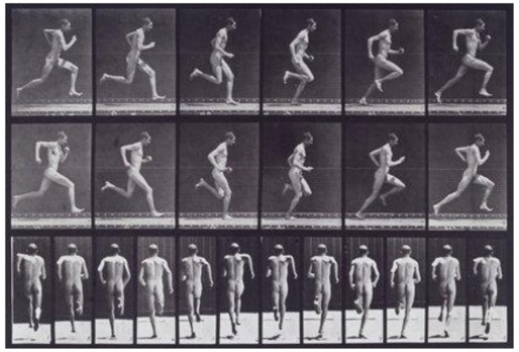
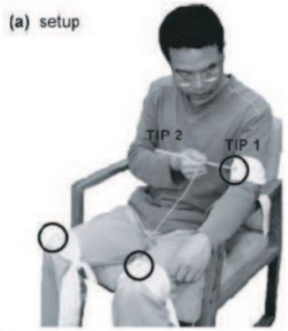


Modeling Human Behavior with Optimal Control



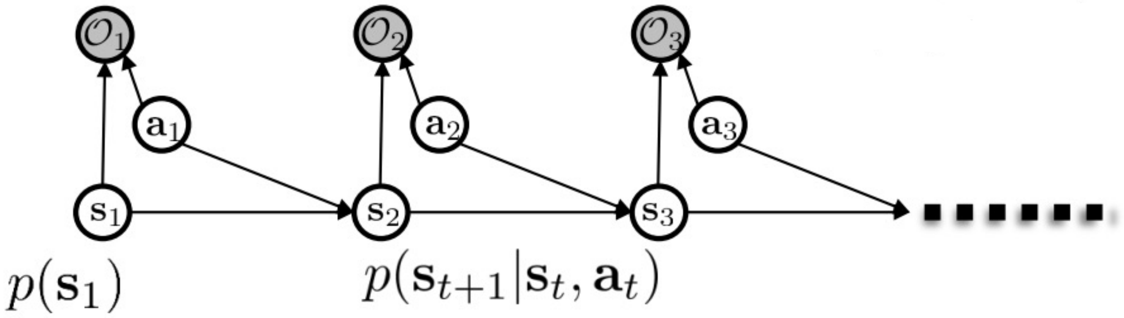
Muybridge (c. 1870)

Mombaur et al. '09



Li & Todorov '06

Ziebart '08



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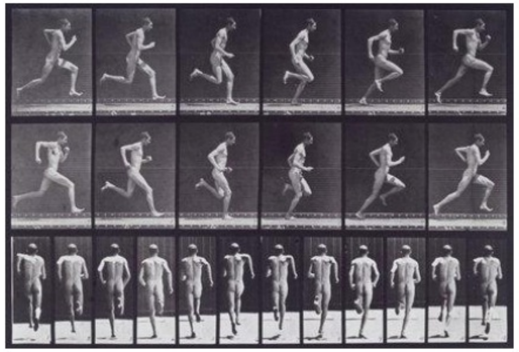

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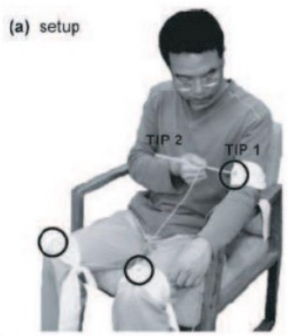

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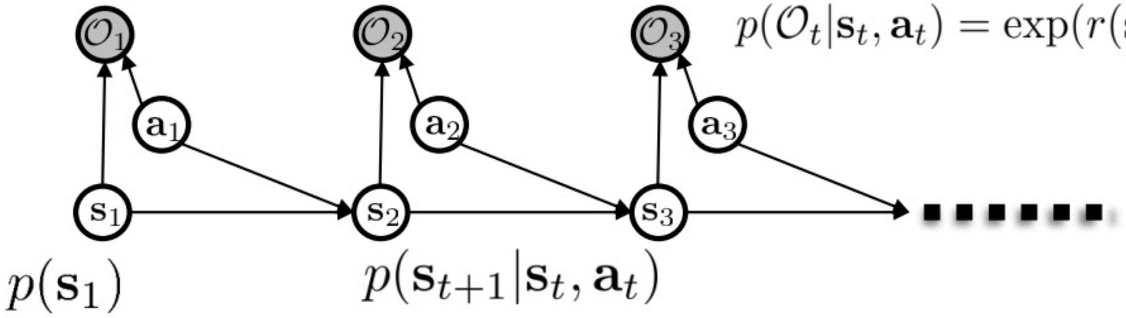
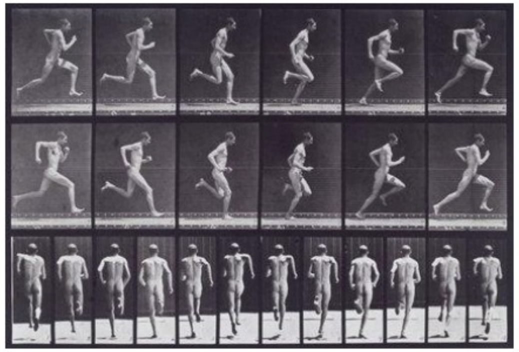


$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

Modeling Human Behavior with Optimal Control



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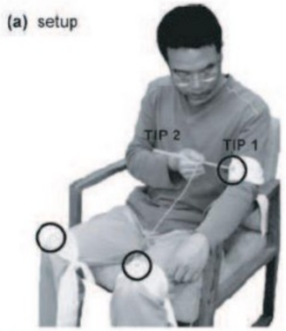

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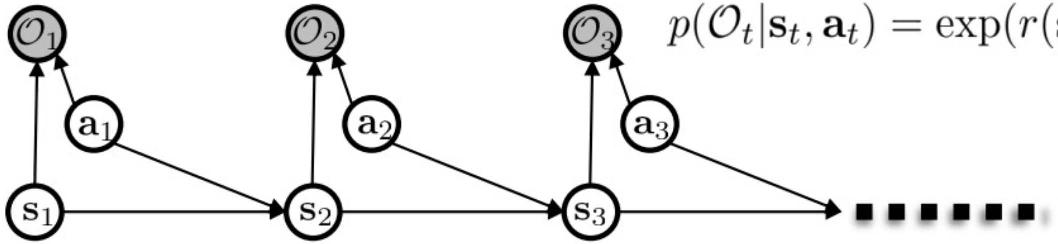
$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

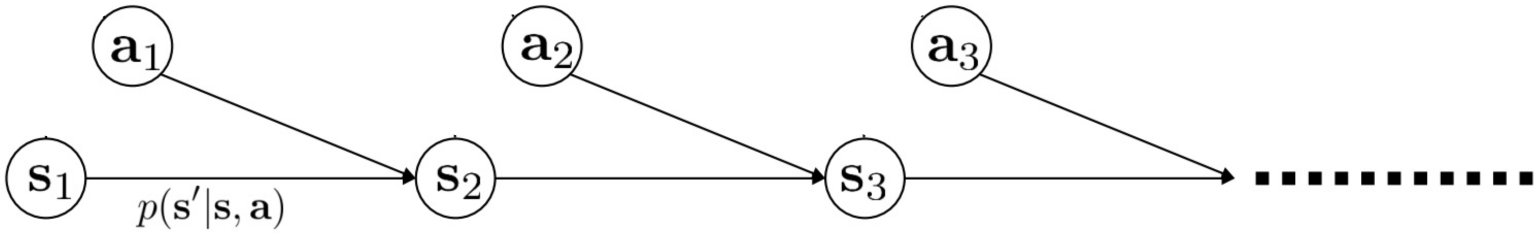
$$p(\mathbf{s}_{1:T}, \mathbf{a}_{1:T}|\mathcal{O}_{1:T})$$



$$p(\mathbf{s}_1) \qquad p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

A probabilistic graphical model of decision making

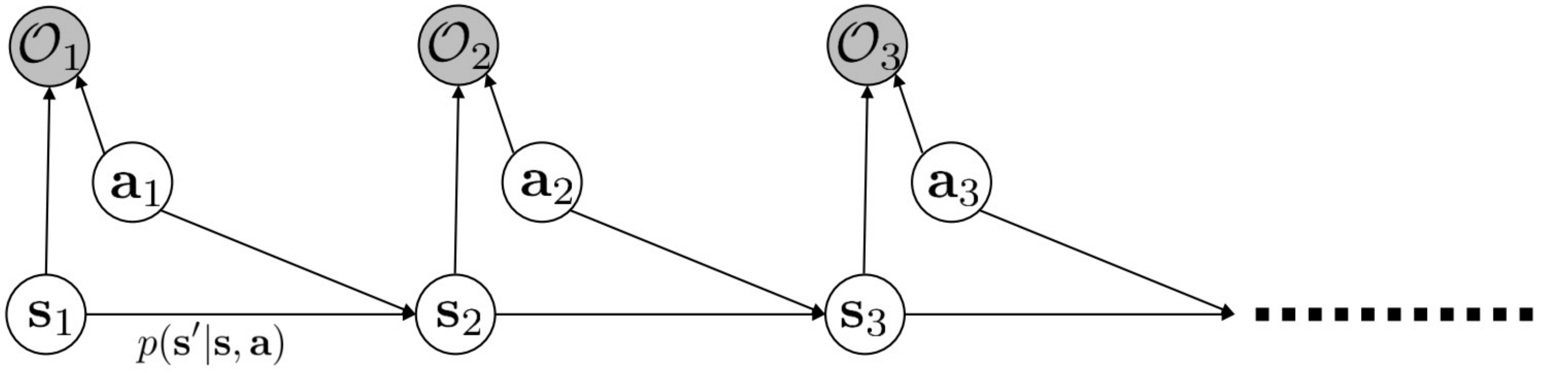
$$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$$



A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

$p(\tau|\mathcal{O}_{1:T})$ $p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$



A probabilistic graphical model of decision making

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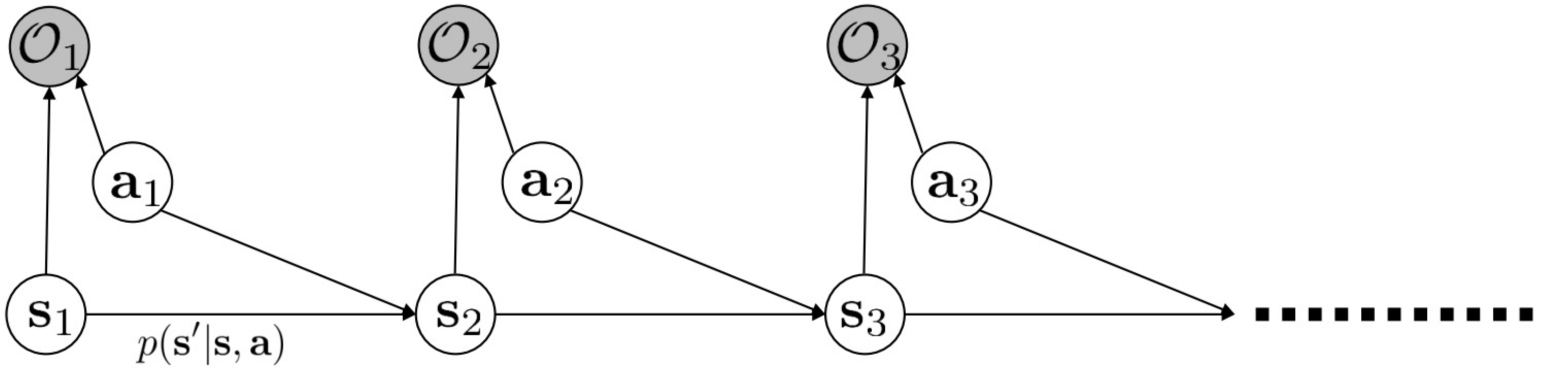
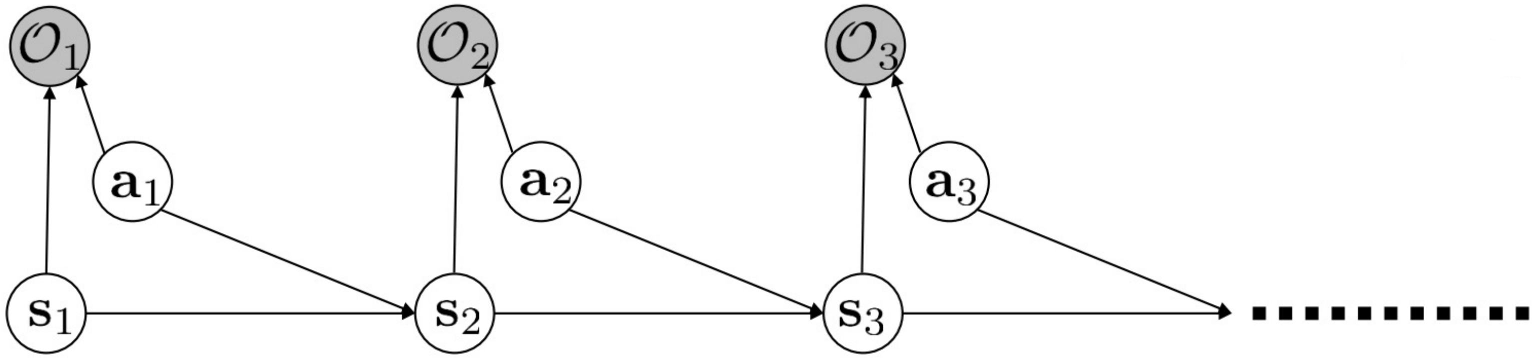
$$p(\tau|\mathcal{O}_{1:T}) = \frac{p(\tau, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})}$$

$$\propto p(\tau) \prod_t \exp(r(\mathbf{s}_t, \mathbf{a}_t)) = p(\tau) \exp\left(\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right)$$



Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$



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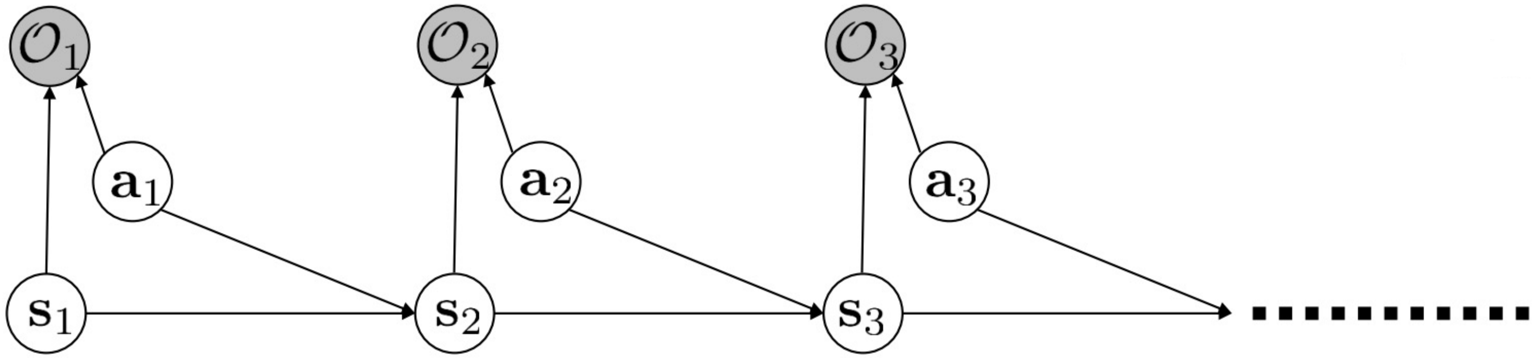
$$p(\mathcal{O}_t|\mathbf{s}_t,\mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t,\mathbf{a}_t))$$

reward parameters



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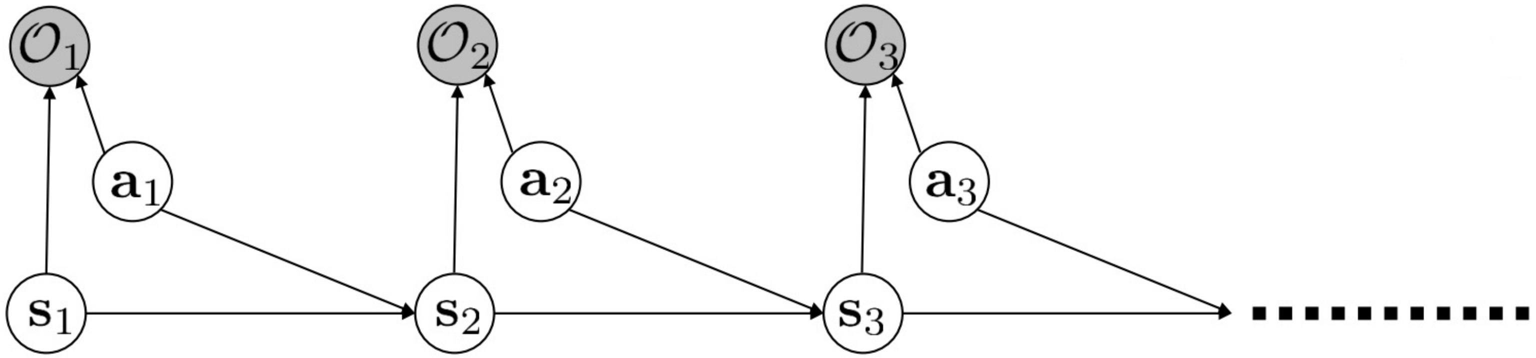
$$p(\tau|\mathcal{O}_{1:T}, \psi)$$



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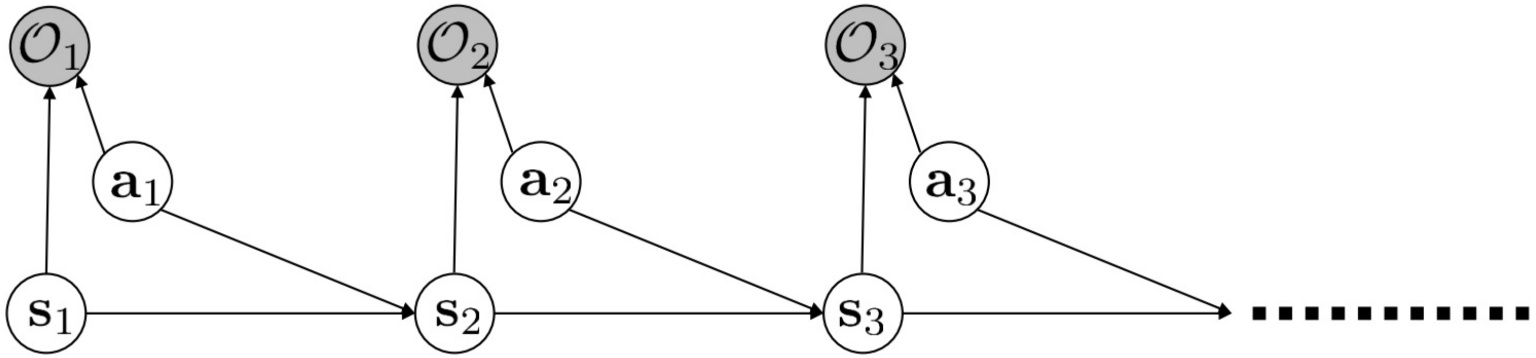

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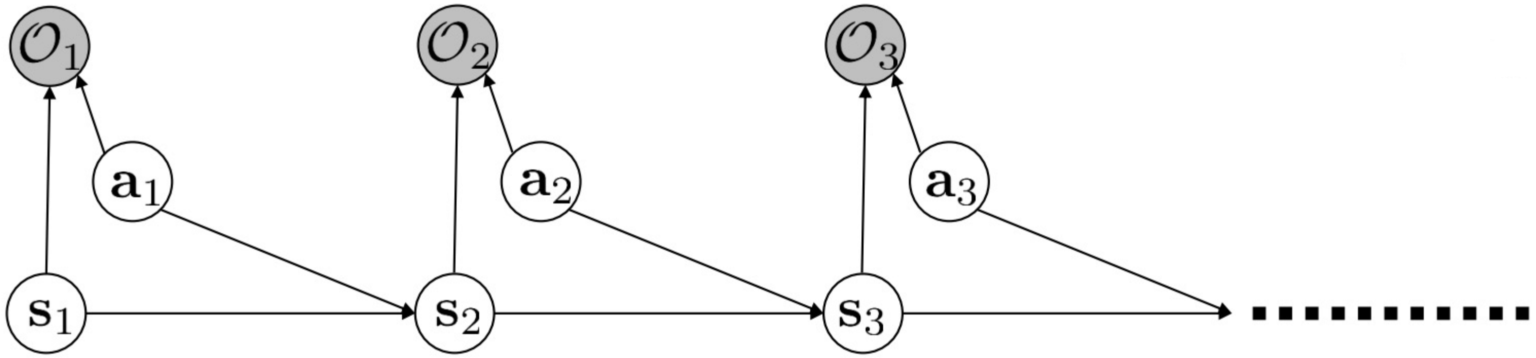
samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$



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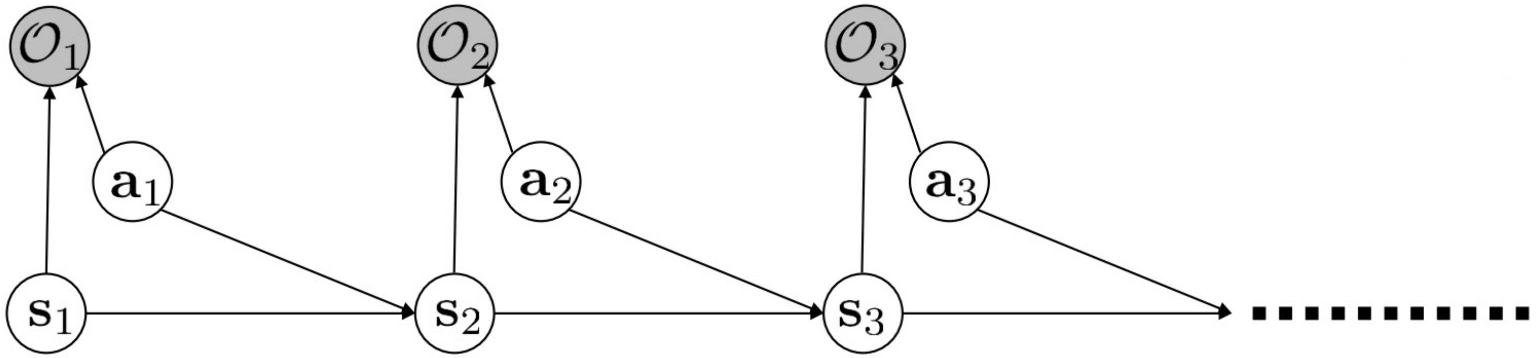
maximum likelihood learning



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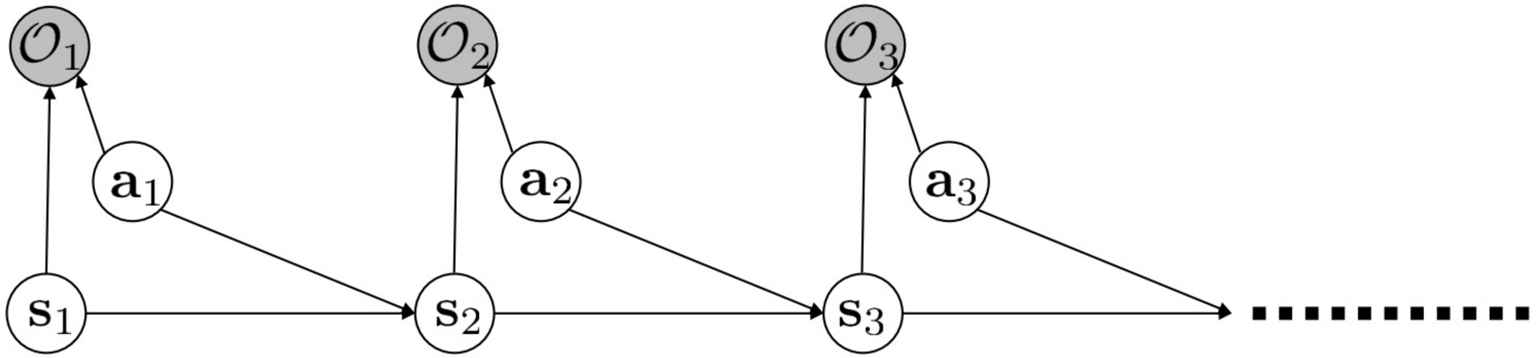

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can ignore (independent of ψ)

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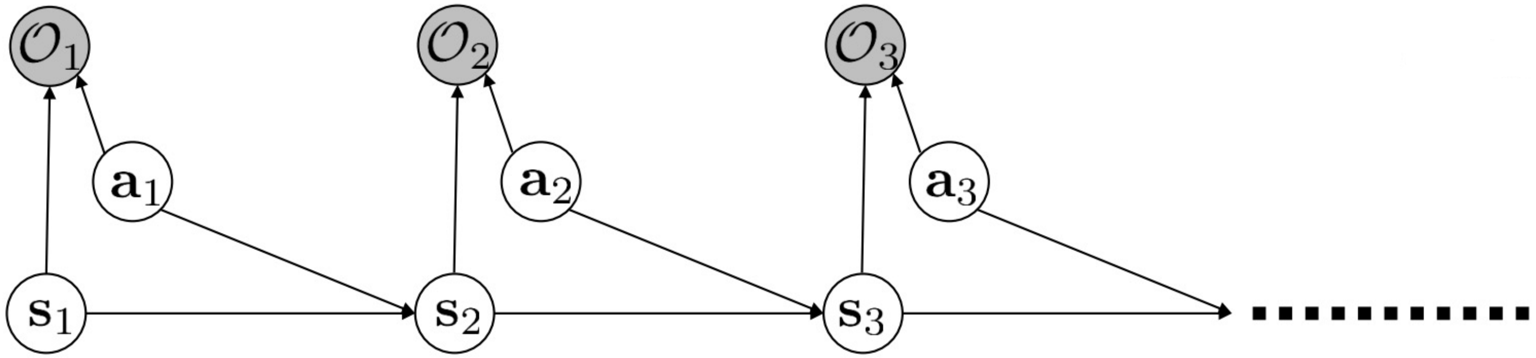

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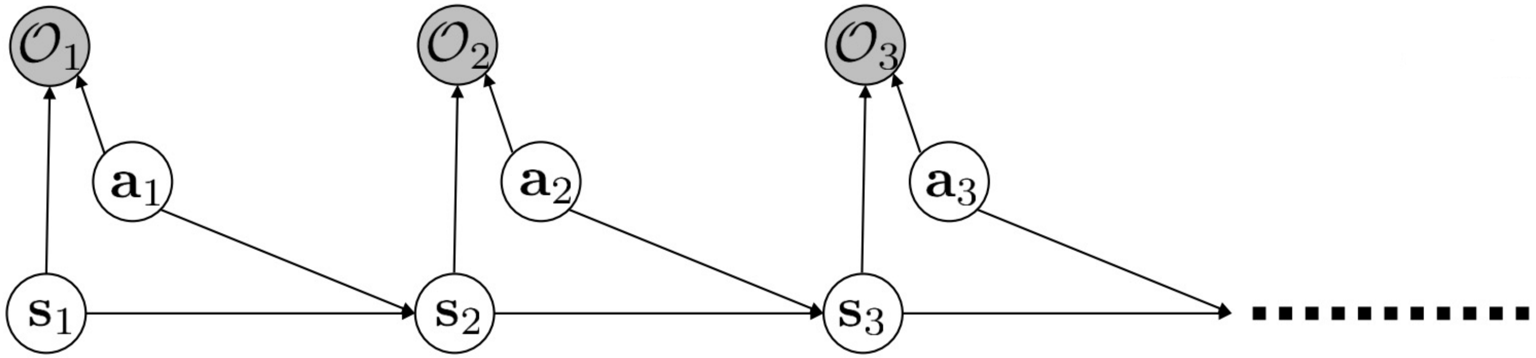

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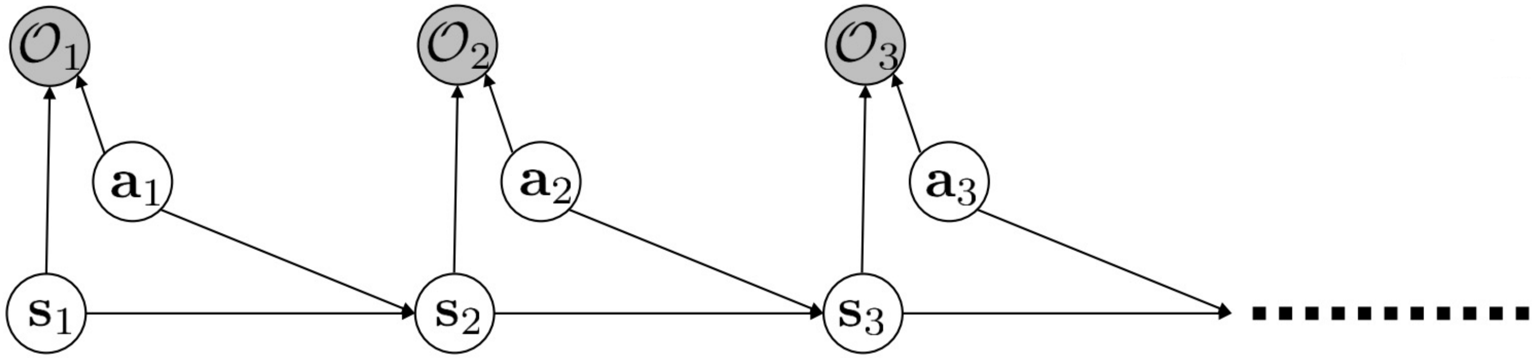
partition function
(the hard part)



The IRL Partiotion Function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z$$

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$$Z = \int p(\tau) \exp(r_{\psi}(\tau)) d\tau$$

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$$\nabla_\psi \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{Z} \int p(\tau) \exp(r_\psi(\tau)) \nabla_\psi r_\psi(\tau) d\tau$$

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estimate with expert samples

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estimate with expert samples

soft optimal policy under current reward

Unknown Dynamics & Large State/Action Spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]$$

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idea: learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

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estimate with expert samples

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$$J(\theta) = \sum_t E_{\pi(\mathbf{s}_t, \mathbf{a}_t)}[r_\psi(\mathbf{s}_t, \mathbf{a}_t)] + E_{\pi(\mathbf{s}_t)}[\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_t))]$$

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estimate with expert samples — soft optimal policy under current reward

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then run this policy to sample $\{\tau_j\}$

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sum over expert samples

sum over policy samples

More Efficient Sample-Based Updates

$$\nabla_\psi \mathcal{L} \approx \underbrace{\frac{1}{N}\sum_{i=1}^{N}}_{\text{sum over expert samples}} \nabla_\psi r_\psi(\tau_i) - \underbrace{\frac{1}{M}\sum_{j=1}^{M}}_{\text{sum over policy samples}} \nabla_\psi r_\psi(\tau_j)$$

learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

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sum over expert samples — sum over policy samples

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looks expensive! what if we use "lazy" policy optimization?

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sum over expert samples

sum over policy samples

improve ~~learn~~ $p(\mathbf{a}_t | \mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

then run this policy to sample $\{\tau_j\}$

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improve (a little) ~~learn~~ $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

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problem: estimator is now biased! wrong distribution!

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sum over expert samples | sum over policy samples

improve (a little) ~~learn~~ $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm
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looks expensive! what if we use "lazy" policy optimization?

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solution 1: use importance sampling

More Efficient Sample-Based Updates

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solution 1: use importance sampling

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

More Efficient Sample-Based Updates

$$\nabla_\psi \mathcal{L} \approx \underbrace{\frac{1}{N}\sum_{i=1}^{N}}_{\text{sum over expert samples}} \nabla_\psi r_\psi(\tau_i) - \underbrace{\frac{1}{M}\sum_{j=1}^{M}}_{\text{sum over policy samples}} \nabla_\psi r_\psi(\tau_j)$$

improve (a little) ~~learn~~ $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm
then run this policy to sample $\{\tau_j\}$

looks expensive! what if we use "lazy" policy optimization?

problem: estimator is now biased! wrong distribution!

solution 1: use importance sampling

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j) \qquad w_j = \frac{p(\tau)\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

Importance Sampling

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j) \qquad w_j = \frac{p(\tau)\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

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$$w_j = \frac{p(\tau)\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

$$\frac{p(\mathbf{s}_1)\prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)\exp(r_\psi(\mathbf{s}_t,\mathbf{a}_t))}{p(\mathbf{s}_1)\prod_t p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)\pi(\mathbf{a}_t|\mathbf{s}_t)}$$

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$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

$$w_j = \frac{p(\tau)\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

$$\frac{\cancel{p(\mathbf{s}_1)} \prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)} \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{\cancel{p(\mathbf{s}_1)} \prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)} \pi(\mathbf{a}_t|\mathbf{s}_t)}$$

Importance Sampling

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

$$w_j = \frac{p(\tau)\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

$$\frac{\cancel{p(\mathbf{s}_1)}\prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}\exp(r_\psi(\mathbf{s}_t,\mathbf{a}_t))}{\cancel{p(\mathbf{s}_1)}\prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)}\pi(\mathbf{a}_t|\mathbf{s}_t)}$$

$$= \frac{\exp(\sum_t r_\psi(\mathbf{s}_t,\mathbf{a}_t))}{\prod_t \pi(\mathbf{a}_t|\mathbf{s}_t)}$$

Importance Sampling

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

$$w_j = \frac{p(\tau)\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

$$\uparrow$$

$$\frac{\cancel{p(\mathbf{s}_1)}\prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}\exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{\cancel{p(\mathbf{s}_1)}\prod_t \cancel{p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}\pi(\mathbf{a}_t|\mathbf{s}_t)}$$

$$= \frac{\exp(\sum_t r_\psi(\mathbf{s}_t, \mathbf{a}_t))}{\prod_t \pi(\mathbf{a}_t|\mathbf{s}_t)}$$

each policy update w.r.t. r_ψ brings us closer to the target distribution!

Guided Cost Learning Algorithm (Finn et al. ICML ’16)

initial
policy π

human
demonstrations

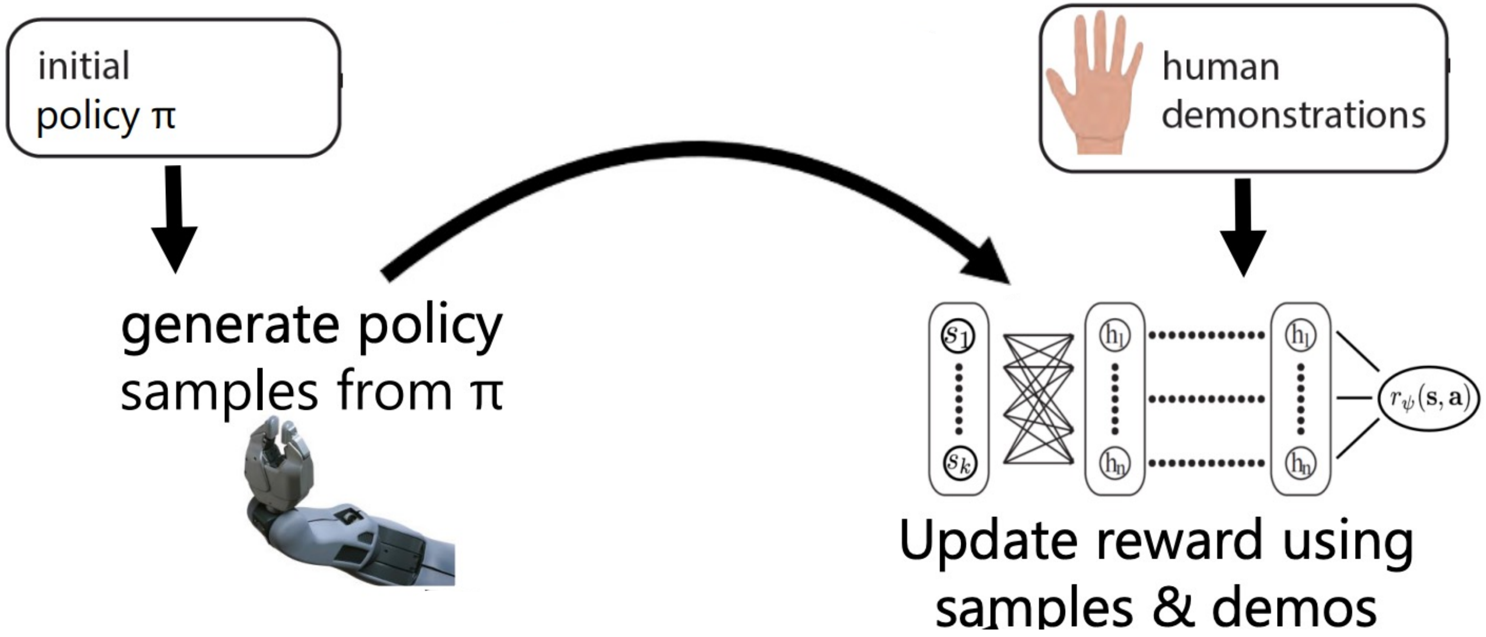
Guided Cost Learning Algorithm (Finn et al. ICML ’16)

initial
policy π

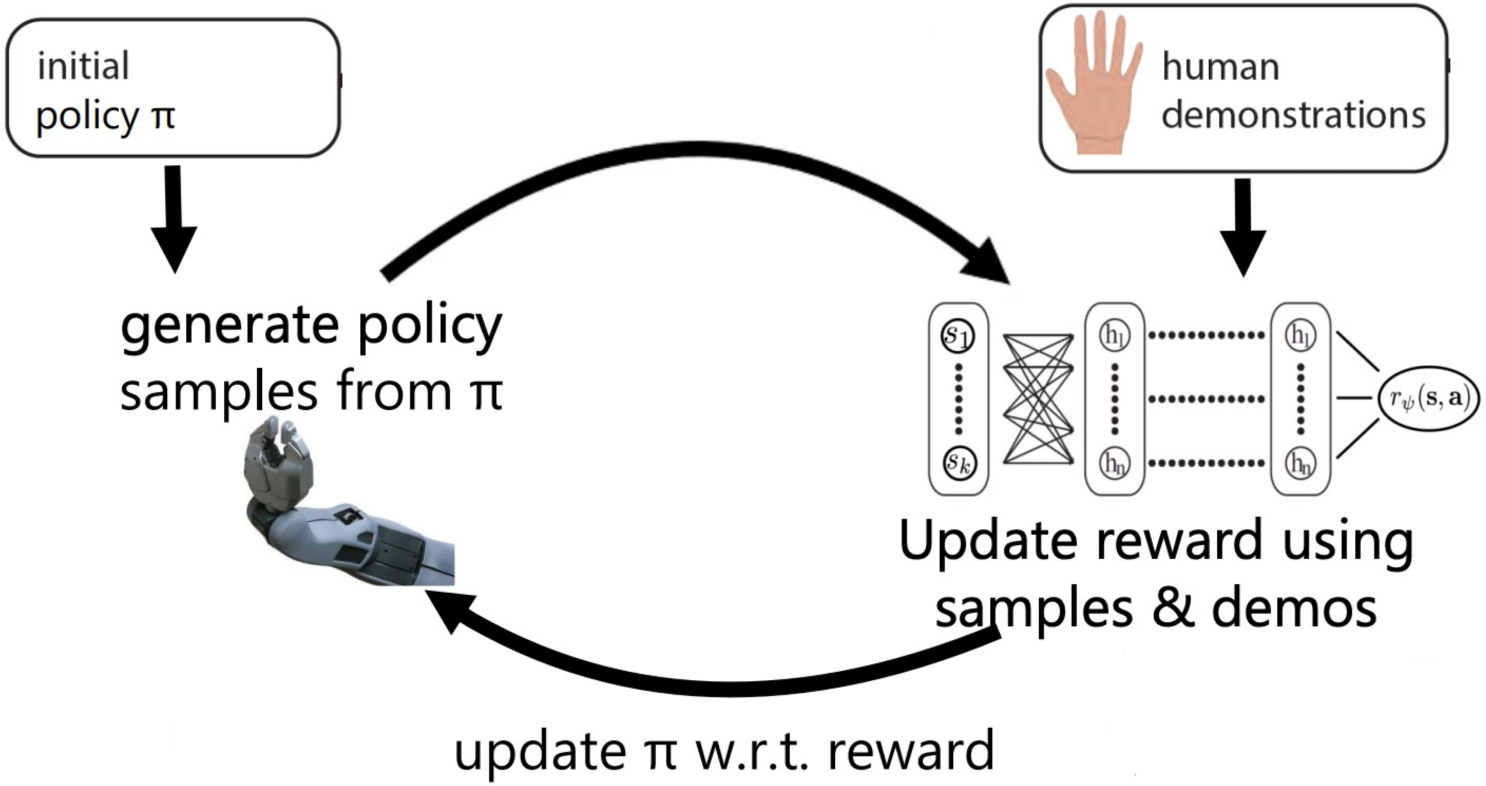
human
demonstrations

generate policy
samples from π

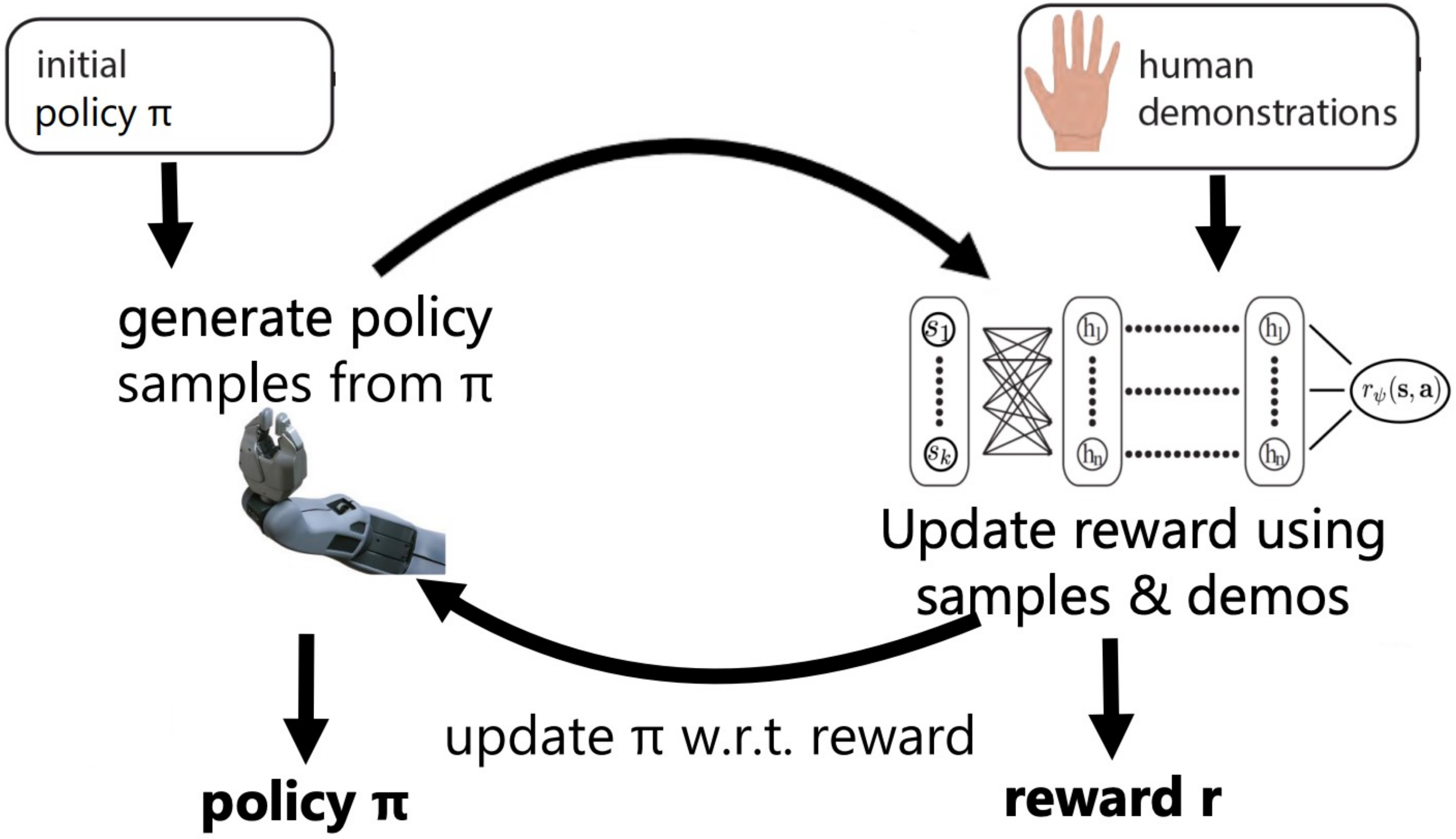
Guided Cost Learning Algorithm (Finn et al. ICML '16)



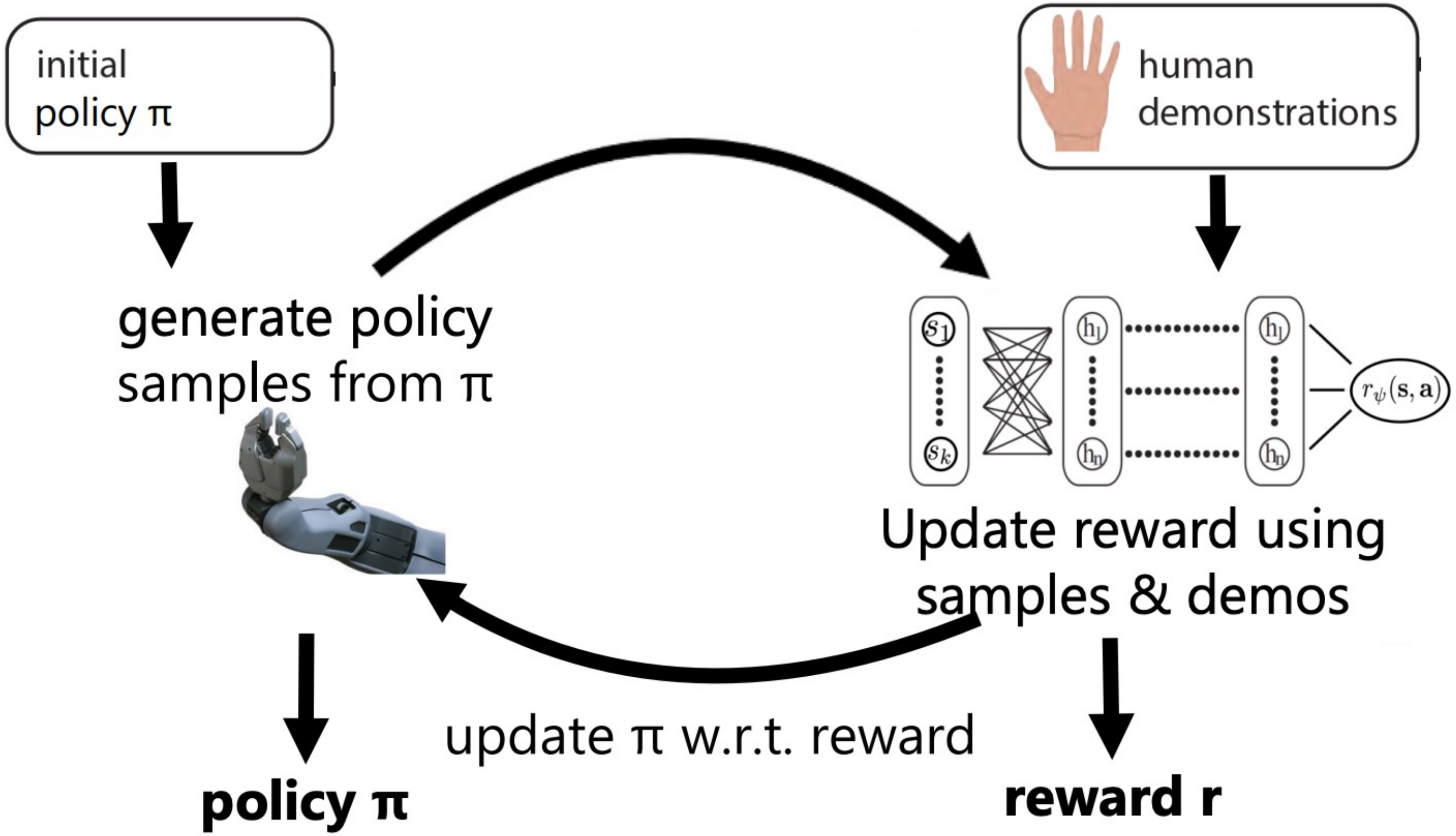
Guided Cost Learning Algorithm (Finn et al. ICML ’16)



Guided Cost Learning Algorithm (Finn et al. ICML ’16)

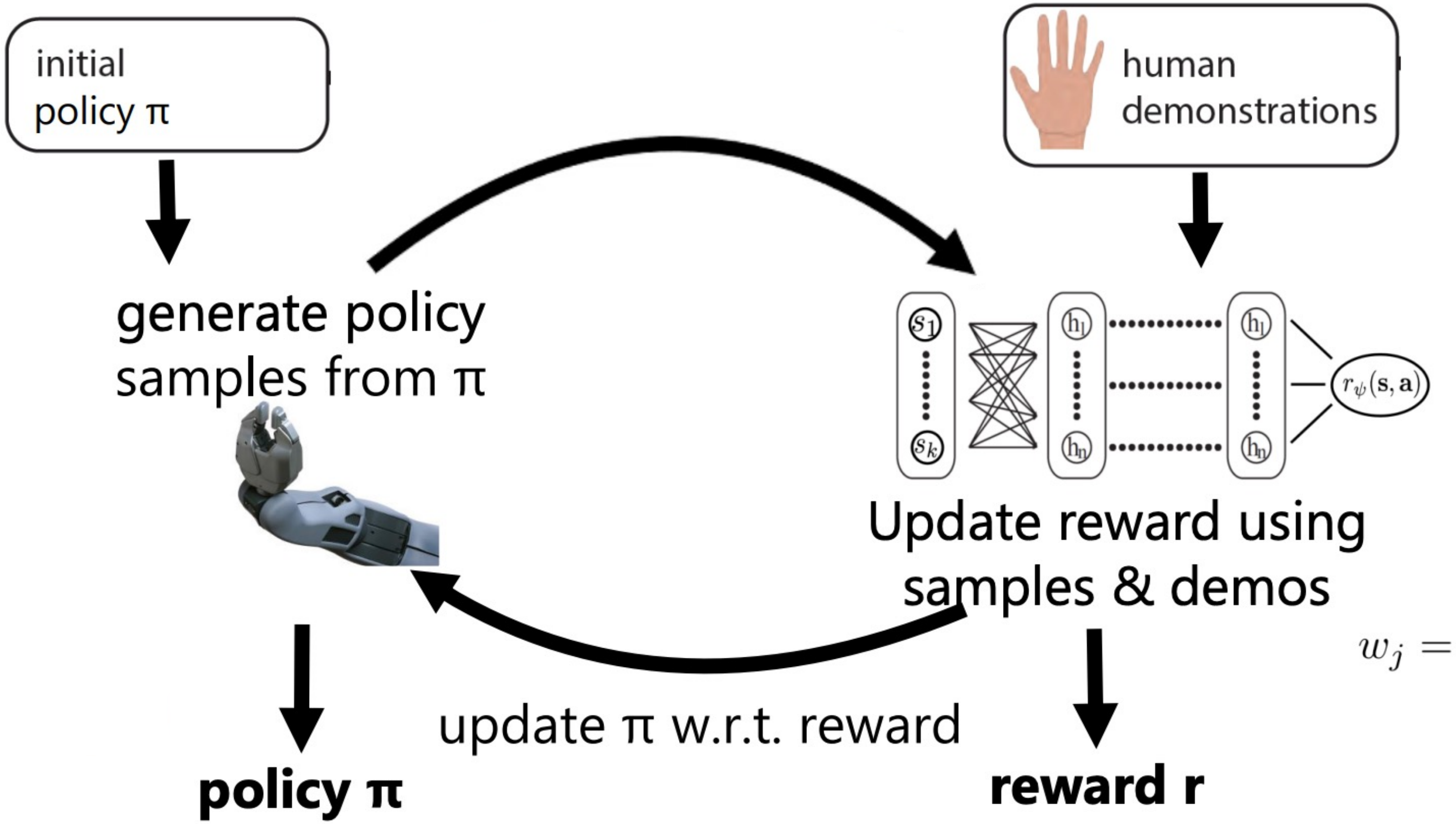


Guided Cost Learning Algorithm (Finn et al. ICML '16)



$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j}\sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

Guided Cost Learning Algorithm (Finn et al. ICML ’16)



$$w_j = \frac{\exp(r_\psi(\tau_j))}{\pi(\tau_j)}$$

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{\sum_j w_j} \sum_{j=1}^{M} w_j \nabla_\psi r_\psi(\tau_j)$$

Suggested Reading on Inverse RL

Classic Papers:
Abbeel & Ng ICML '04. *Apprenticeship Learning via Inverse Reinforcement Learning*. Good introduction to inverse reinforcement learning
Ziebart et al. AAAI '08. *Maximum Entropy Inverse Reinforcement Learning*. Introduction to probabilistic method for inverse reinforcement learning

Modern Papers:
Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions
Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions
Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks
Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning