Computer Engineering Department

Inverse Reinforcement Learning

Mohammad Hossein Rohban, Ph.D.

Spring 2025

Slides are adopted from CS 285, UC Berkeley.

Lecture Outline

1. So far: manually design reward function to define a task
2. What if we want to *learn* the reward function from observing an expert, and then use reinforcement learning?
3. Apply approximate optimality model from last time, but now learn the reward!

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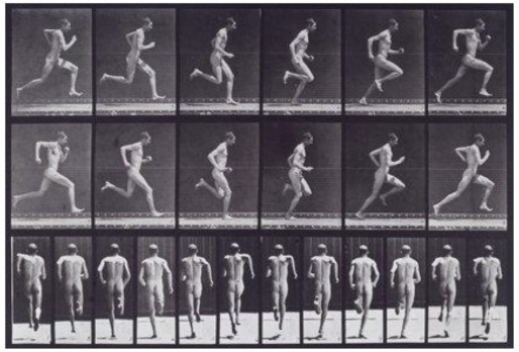
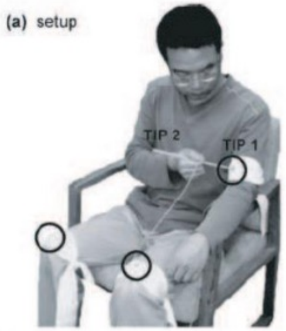
- Goals:
  - Understand the inverse reinforcement learning problem definition
  - Understand how probabilistic models of behavior can be used to derive inverse reinforcement learning algorithms
  - Understand a few practical inverse reinforcement learning algorithms we can use

Modeling Human Behavior with Optimal Control


Muybridge (c. 1870)


Mombaur et al. '09



Li & Todorov '06


Ziebart '08

$$\mathbf{a}_1, \ldots, \mathbf{a}_T = \arg \max_{\mathbf{a}_1, \ldots, \mathbf{a}_T} \sum_{t=1}^{T} r(\mathbf{s}_t, \mathbf{a}_t)$$

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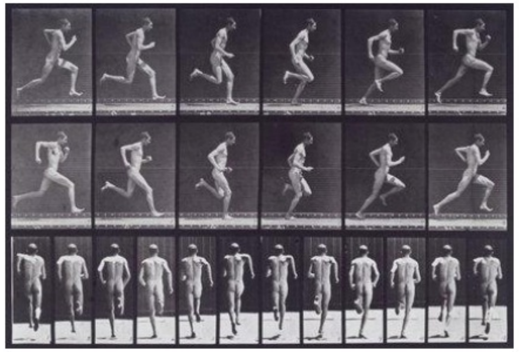
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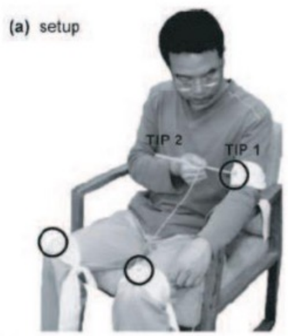

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optimize this to explain the data

$$\pi = \arg\max_{\pi} E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t), \mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)}[r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$\mathbf{a}_t \sim \pi(\mathbf{a}_t|\mathbf{s}_t)$$

Imitation learning vs RL perspective

The imitation learning perspective

Standard imitation learning:

- copy the *actions* performed by the expert
- no reasoning about outcomes of actions

Human imitation learning:

- copy the *intent* of the expert
- might take very different actions!

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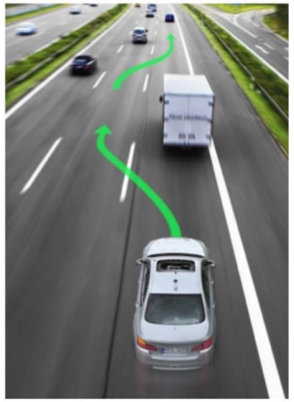
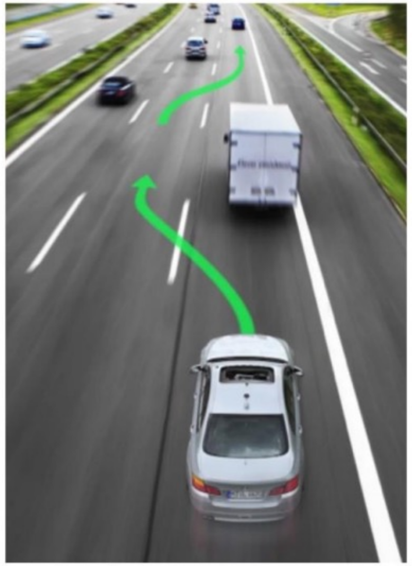


what is the reward?

Inverse Reinforcement Learning

Infer **reward functions** from **demonstrations**



$r(\mathbf{s}, \mathbf{a})$

Inverse Reinforcement Learning

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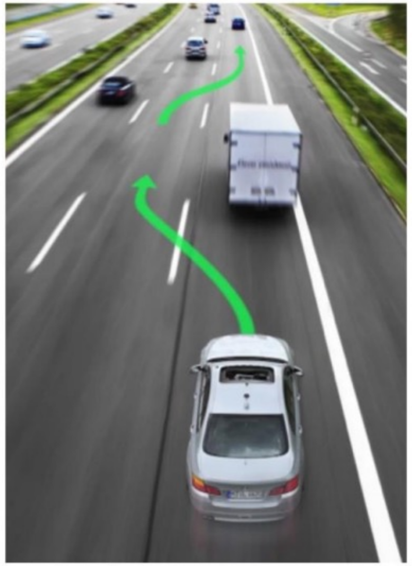
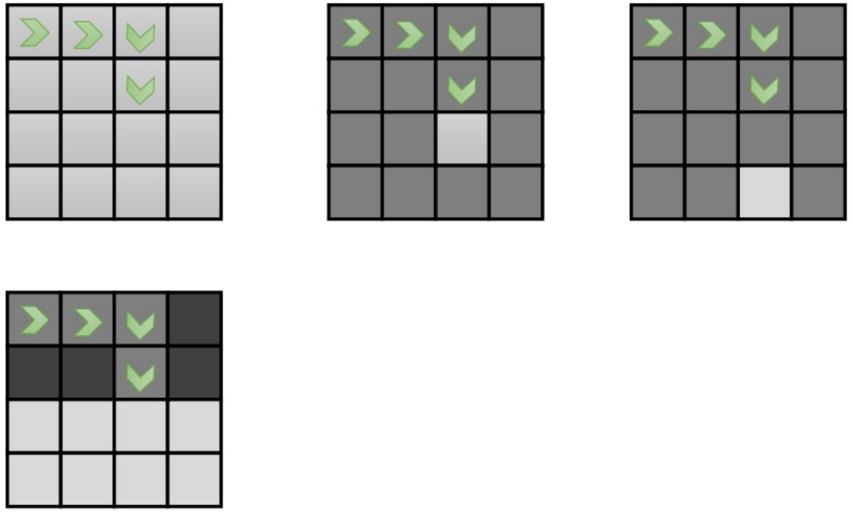


$$r(\mathbf{s}, \mathbf{a})$$

by itself, this is an **underspecified** problem

many reward functions can explain the **same** behavior



Inverse Reinforcement Learning Formulation

"forward" reinforcement learning

inverse reinforcement learning

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"forward" reinforcement learning

given:

states $\mathbf{s} \in \mathcal{S}$, actions $\mathbf{a} \in \mathcal{A}$

(sometimes) transitions $p(\mathbf{s}'|\mathbf{s}, \mathbf{a})$

inverse reinforcement learning

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reward parameters

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linear reward function:

$$r_\psi(\mathbf{s}, \mathbf{a}) = \sum_i \psi_i f_i(\mathbf{s}, \mathbf{a}) = \psi^T \mathbf{f}(\mathbf{s}, \mathbf{a})$$

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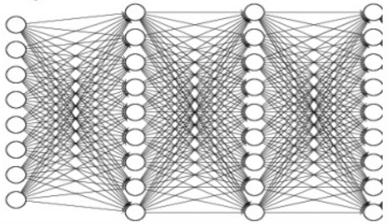
$\mathbf{a}$



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parameters ψ

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reward parameters

...and then use it to learn $\pi^\star(\mathbf{a}|\mathbf{s})$

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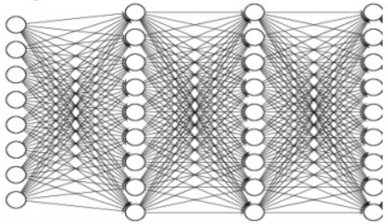

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Feature Matching Inverse RL

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maximum margin principle:

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need to somehow "weight" by similarity between $\pi^\star$ and π

Feature Matching IRL & Maximum Margin

remember the "SVM trick":

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e.g., difference in feature expectations!

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Further reading:

- Abbeel & Ng: Apprenticeship learning via inverse reinforcement learning
- Ratliff et al: Maximum margin planning

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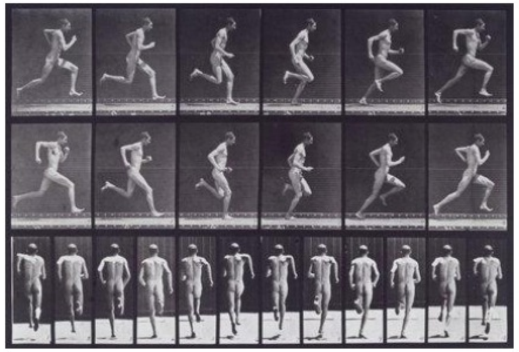

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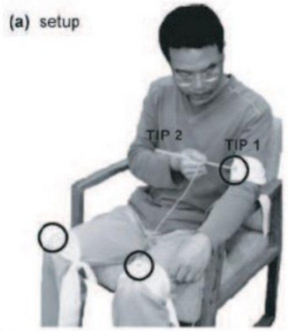

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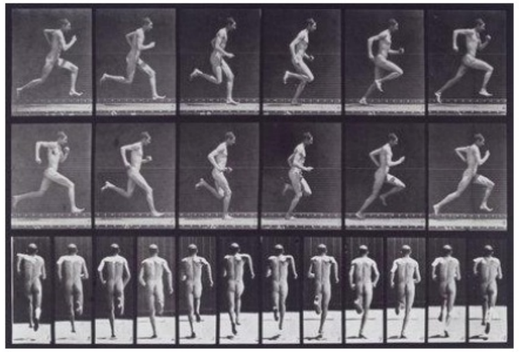

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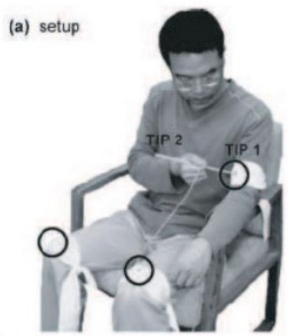

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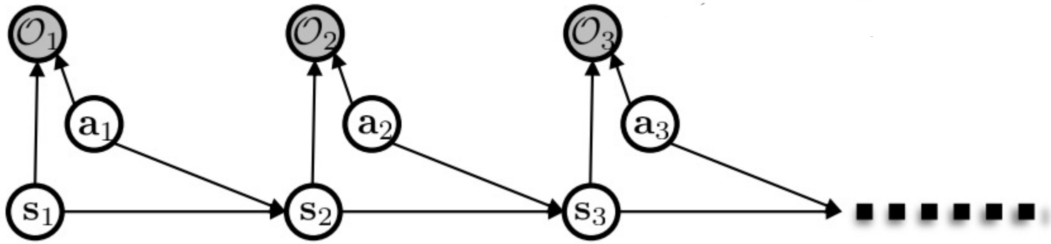

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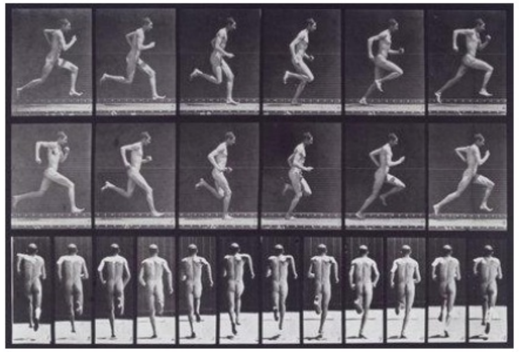

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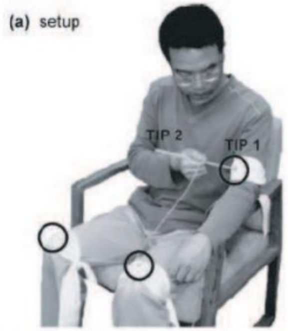

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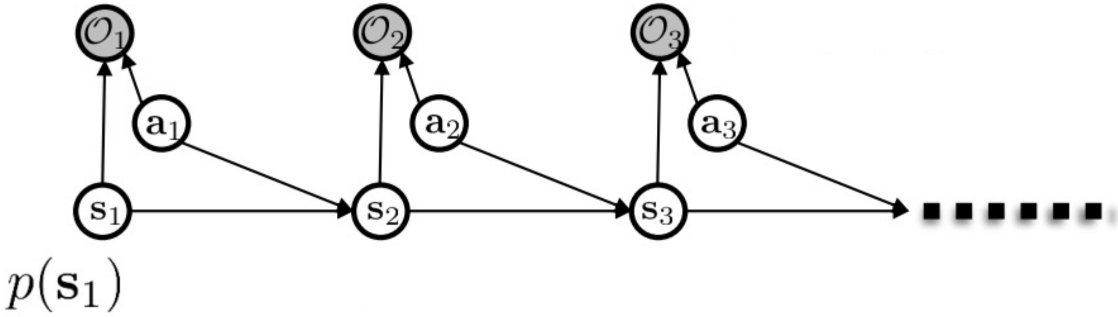
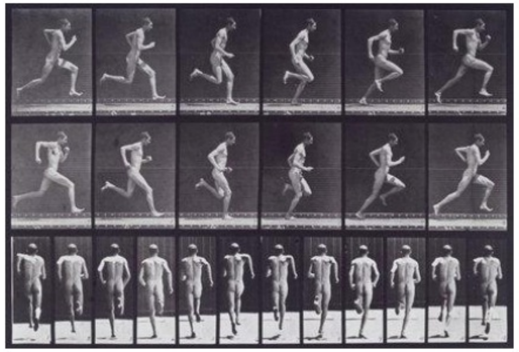
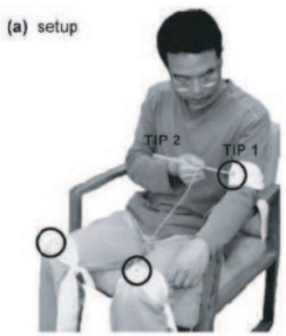


Modeling Human Behavior with Optimal Control



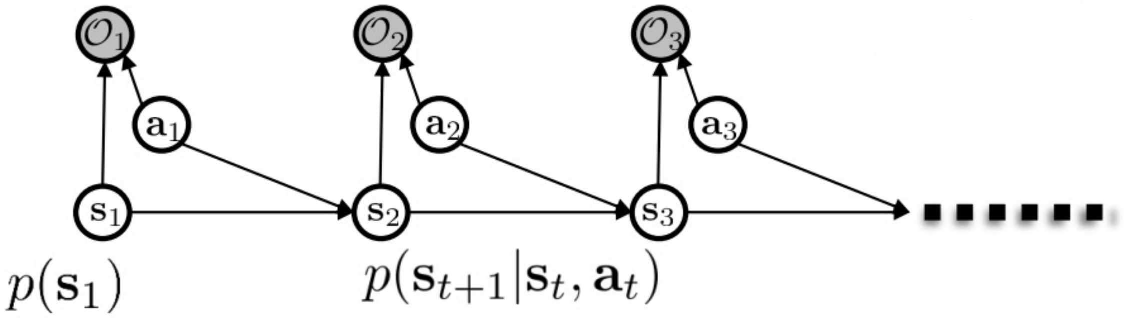
Muybridge (c. 1870)

Mombaur et al. '09

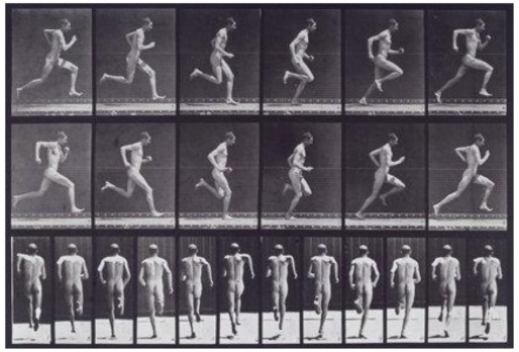


Li & Todorov '06

Ziebart '08



Modeling Human Behavior with Optimal Control



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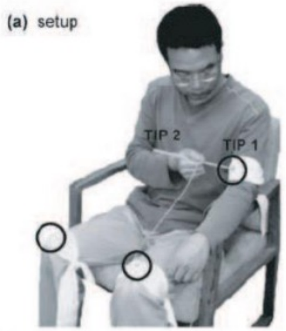

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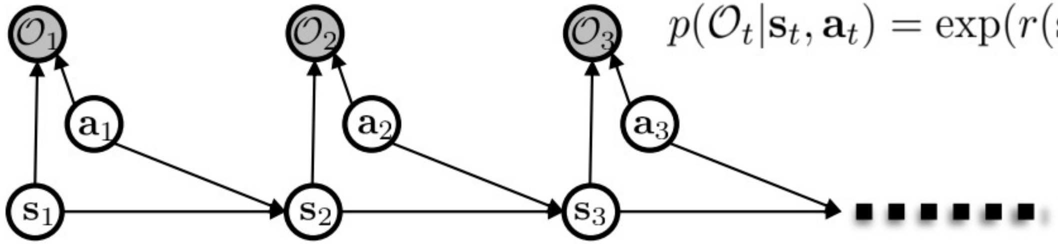
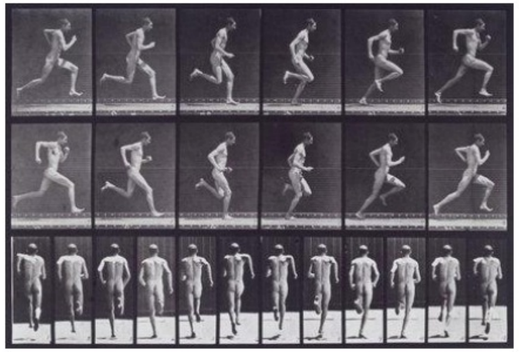


$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$$

$$p(\mathbf{s}_1) \qquad p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

Modeling Human Behavior with Optimal Control



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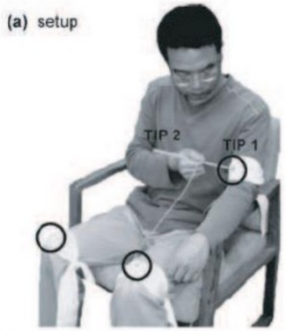
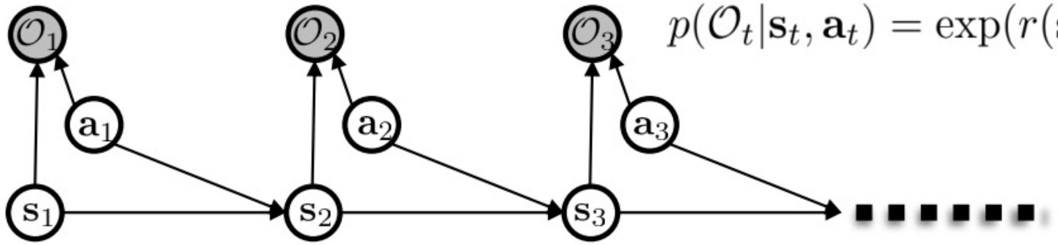


Li & Todorov '06

Ziebart '08

$p(\mathbf{s}_{1:T}, \mathbf{a}_{1:T} | \mathcal{O}_{1:T})$



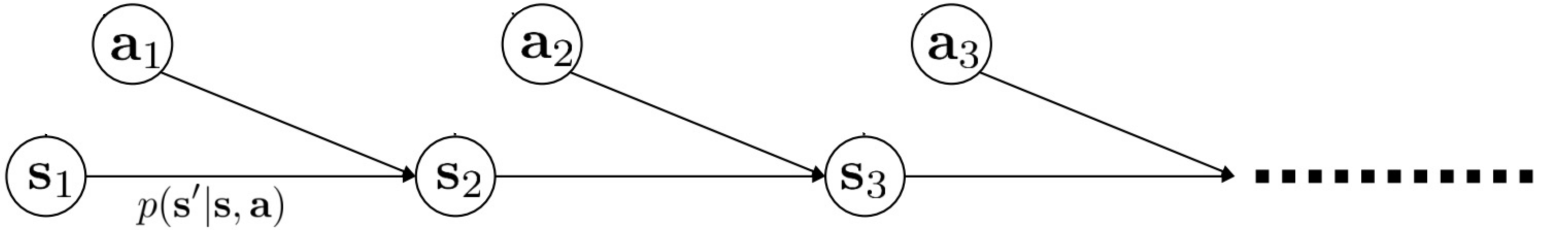
$p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$p(\mathbf{s}_1)$

$p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$

A probabilistic graphical model of decision making

$$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$$

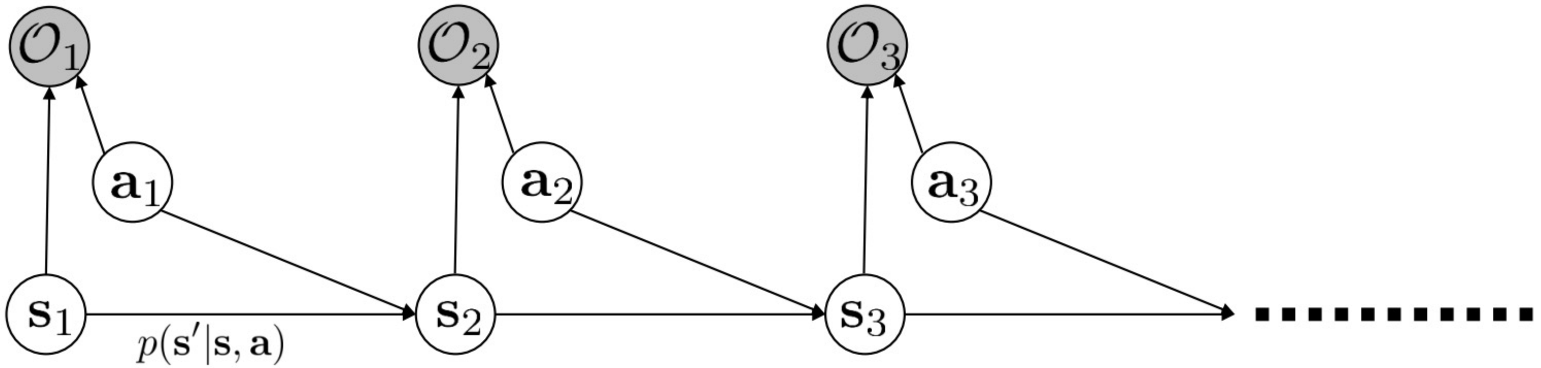


A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

$p(\tau|\mathcal{O}_{1:T})$ $\qquad p(\overset{=1}{\mathcal{O}_t}|\mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$O_{1:T} = \underline{1}$



A probabilistic graphical model of decision making

$p(\underbrace{\mathbf{s}_{1:T}, \mathbf{a}_{1:T}}_{\tau}) = ??$ no assumption of optimal behavior!

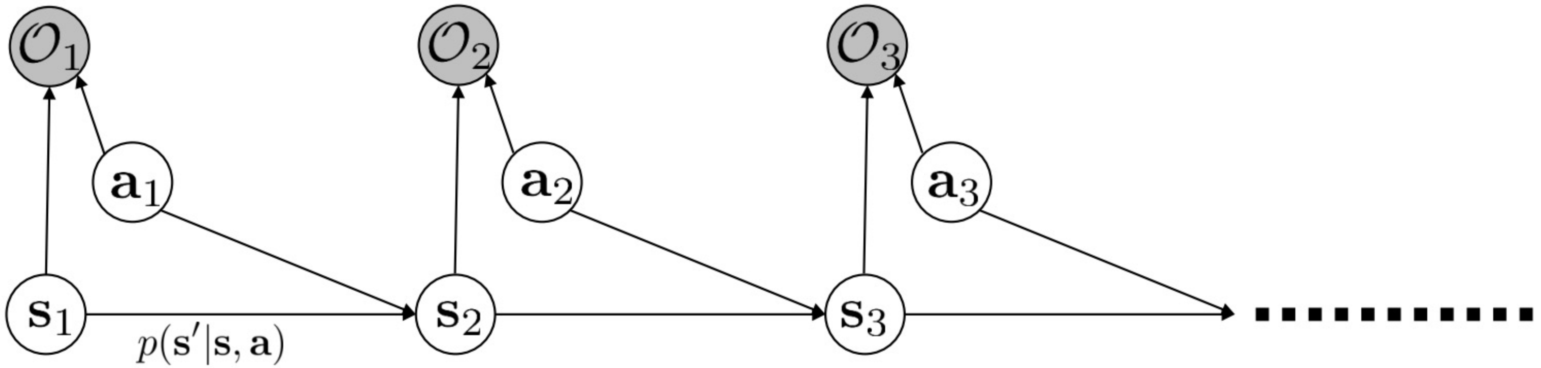
$p(\tau | \mathcal{O}_{1:T})$ $\quad p(\mathcal{O}_t | \mathbf{s}_t, \mathbf{a}_t) = \exp(r(\mathbf{s}_t, \mathbf{a}_t))$

$$p(\tau | \mathcal{O}_{1:T}) = \frac{p(\tau, \mathcal{O}_{1:T})}{p(\mathcal{O}_{1:T})}$$

$$\propto p(\tau) \prod_t \exp(r(\mathbf{s}_t, \mathbf{a}_t)) = p(\tau) \exp\left(\sum_t r(\mathbf{s}_t, \mathbf{a}_t)\right)$$

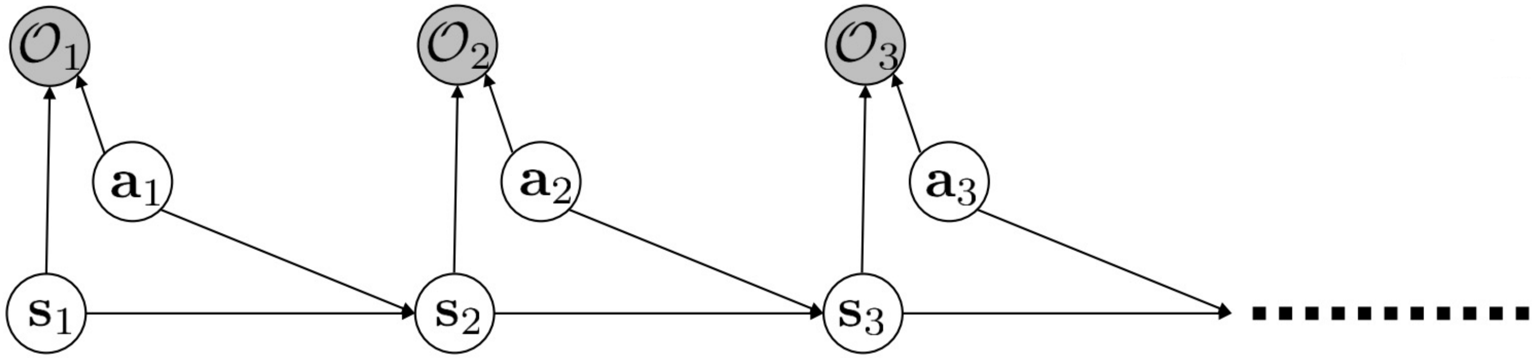
① Soft Opt.

② Convinient Inversion



Learning the optimality variable

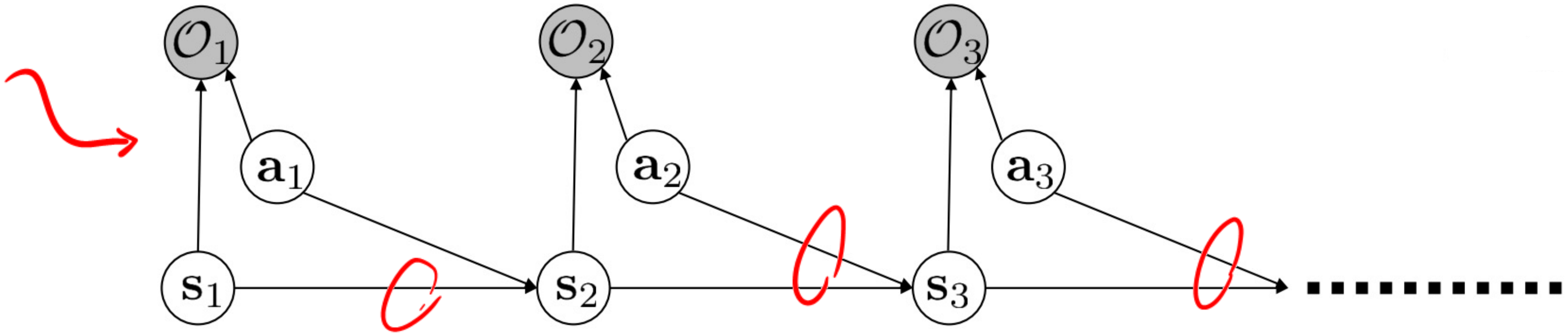
$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$



Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

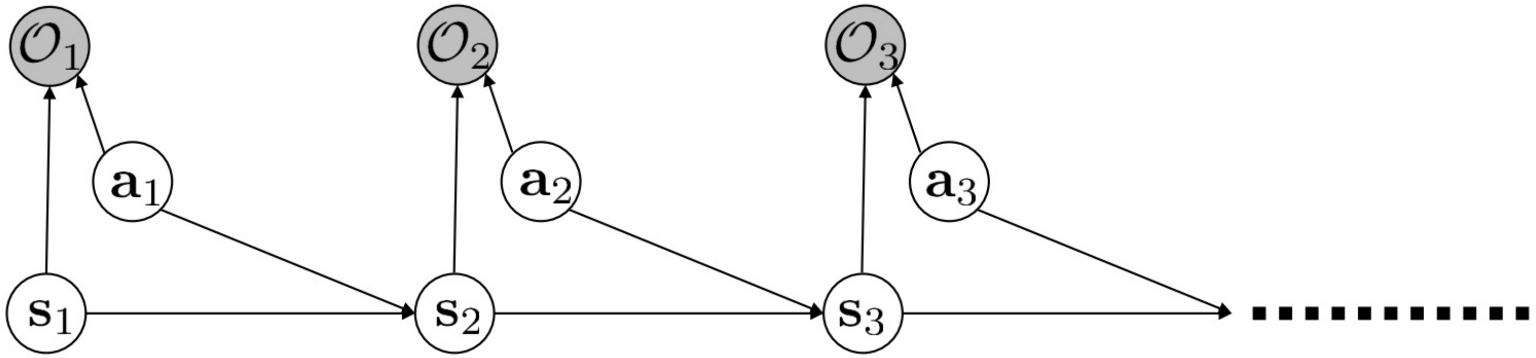
reward parameters



Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t, \psi) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

$$\underbrace{p(\tau|\mathcal{O}_{1:T}, \psi)}_{\text{Likelihood}}$$



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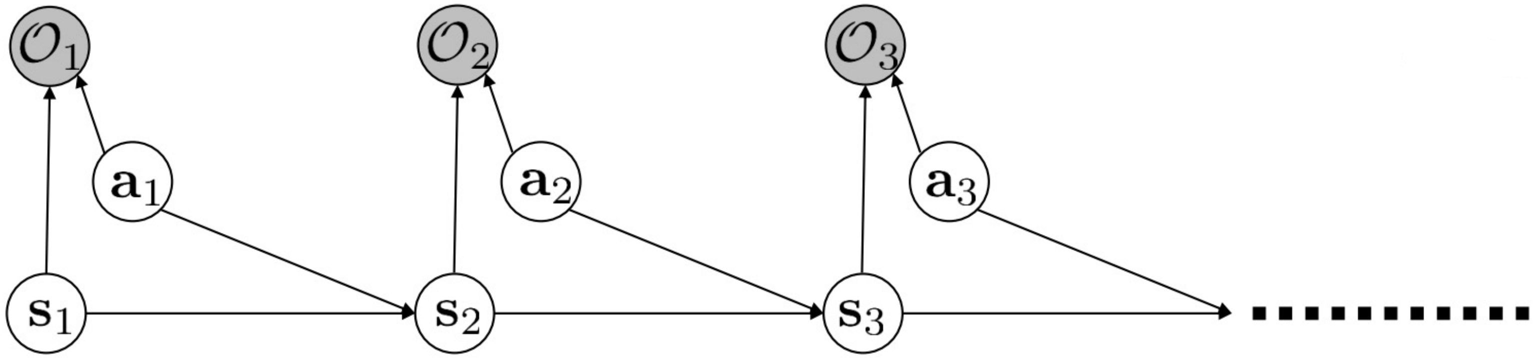

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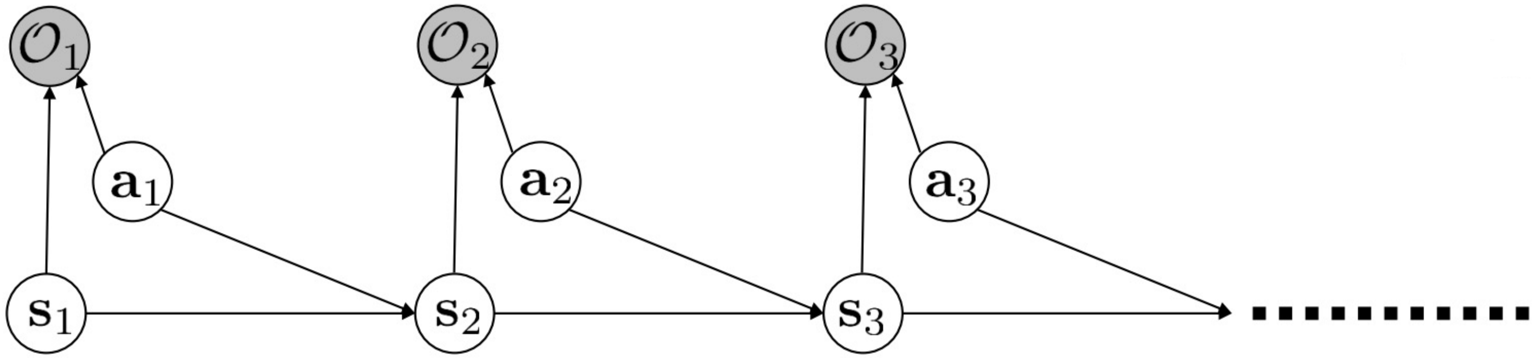
given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$



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$$p(\mathcal{O}_t|\mathbf{s}_t, \mathbf{a}_t, \psi) = \exp(r_\psi(\mathbf{s}_t, \mathbf{a}_t))$$

$$p(\tau|\mathcal{O}_{1:T}, \psi) \propto \underbrace{p(\tau) \exp\left(\sum_t r_\psi(\mathbf{s}_t, \mathbf{a}_t)\right)}_{Z}$$

$\rightarrow Z = \int p(\tau) \exp(\ldots) d\tau$

iid given: $P(\tau_1, \tau_2 \ldots, \tau_N | \psi)$

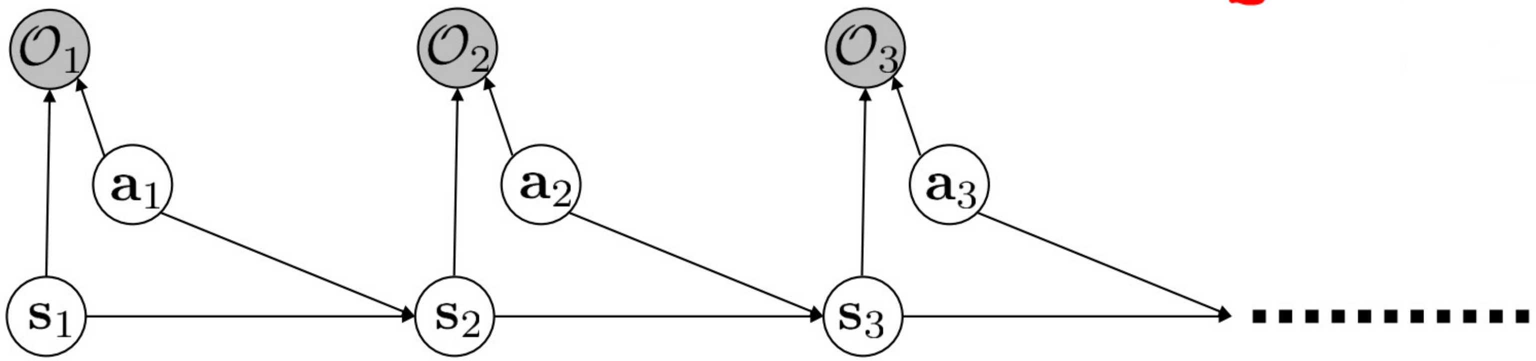
samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$

$= P(\tau_1|\psi) \cdots P(\tau_N|\psi)$

$= \frac{P(\tau_1) \cdots P(\tau_N)}{Z^N} \prod_{i=1}^{N} \exp\left(\sum_t r_\psi(s_t^{(i)}, a_t^{(i)})\right)$

maximum likelihood learning

$\int P(\tau|O_{1:T}, \psi) d\tau = 1$



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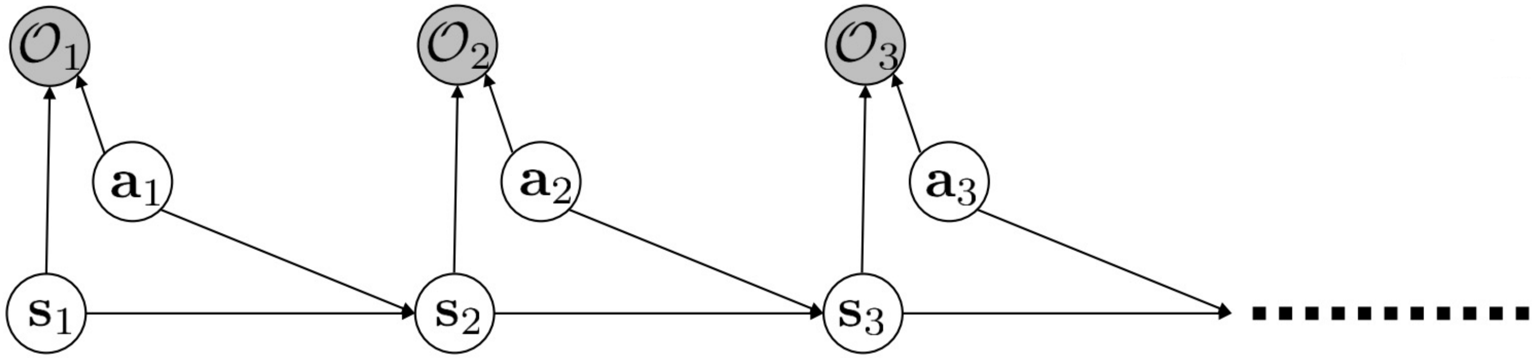
maximum likelihood learning: $\max_\psi \frac{1}{N} \sum_{i=1}^{N} \log p(\tau_i|\mathcal{O}_{1:T}, \psi)$



Learning the optimality variable

$$p(\mathcal{O}_t|\mathbf{s}_t,\mathbf{a}_t,\psi)=\exp(r_\psi(\mathbf{s}_t,\mathbf{a}_t))$$

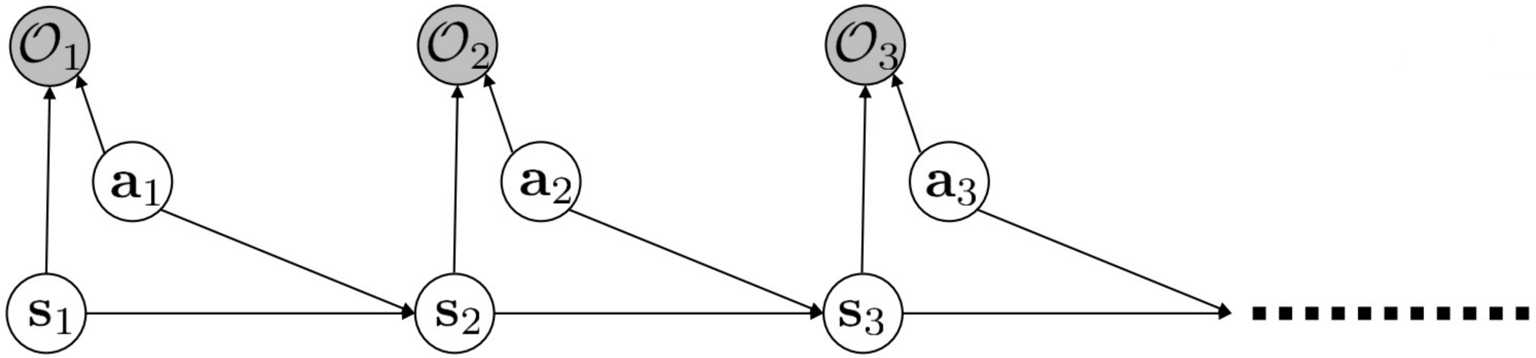
$$p(\tau|\mathcal{O}_{1:T},\psi)\propto \cancel{p(\tau)}\exp\left(\sum_t r_\psi(\mathbf{s}_t,\mathbf{a}_t)\right)$$

can ignore (independent of ψ)

given:

samples $\{\tau_i\}$ sampled from $\pi^\star(\tau)$

maximum likelihood learning: $\max_\psi \frac{1}{N}\sum_{i=1}^N \log p(\tau_i|\mathcal{O}_{1:T},\psi)$



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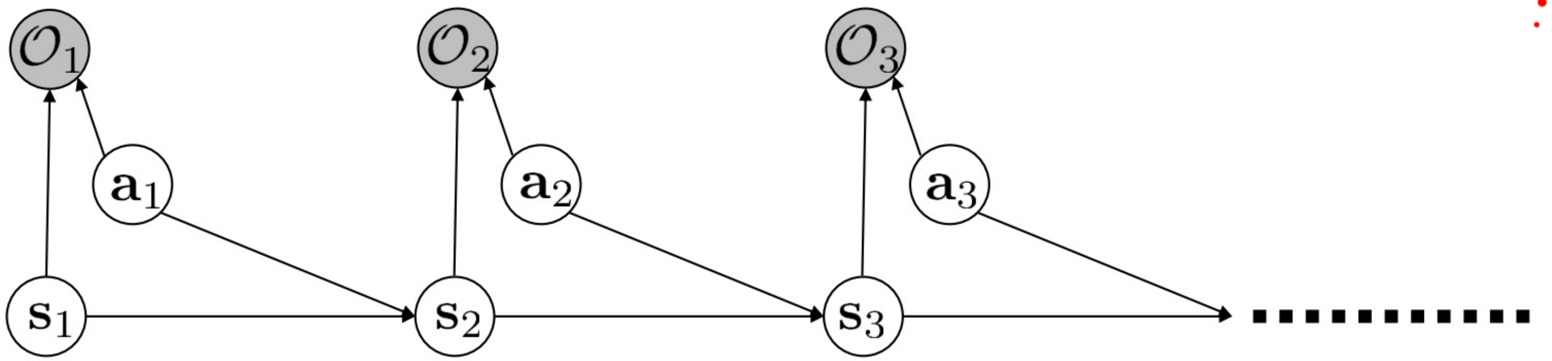
maximum likelihood learning: $$\max_\psi \frac{1}{N}\sum_{i=1}^N \log p(\tau_i|\mathcal{O}_{1:T}, \psi) = \max_\psi \frac{1}{N}\sum_{i=1}^N r_\psi(\tau_i) - \log Z$$

$:= \sum r(s_j, a_j)$

$s_j, a_j \in \tau_i$



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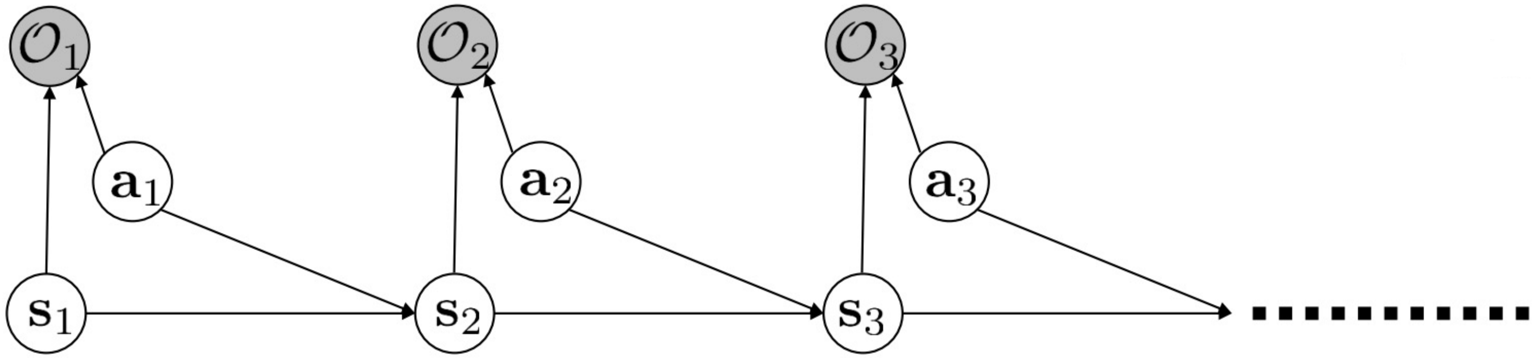
partition function (the hard part)



The IRL Partiotion Function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_{\psi}(\tau_i) - \log Z$$

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$$Z = \int p(\tau) \exp(r_{\psi}(\tau)) d\tau$$

The IRL Partiotion Function

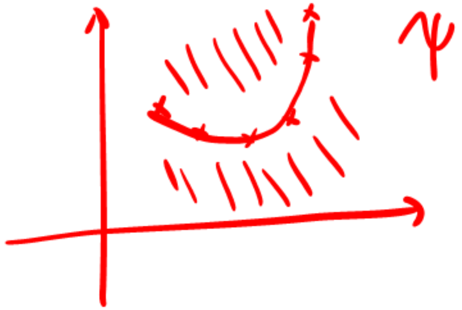
$\tau = (s_1, a_1, s_2, a_2 \dots)$

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_\psi(\tau_i) - \log Z \qquad Z = \int p(\tau) \exp(r_\psi(\tau)) d\tau$$

$p(\tau | O_{1:T}, \psi)$

$$\nabla_\psi \mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{Z} \int p(\tau) \exp(r_\psi(\tau)) \nabla_\psi r_\psi(\tau) d\tau$$

$g_\psi(\tau)$



$$\mathbb{E}_{\tau \sim p(\tau | O_{1:T}, \psi)} \{ \nabla_\psi r_\psi(\tau) \} \approx \frac{1}{M} \sum_{i=1}^{M} \nabla_\psi r_\psi(\tau_i)$$

$\tau_i \sim \text{SAC}(r_\psi)$

The IRL Partiotion Function

$$\max_{\psi} \frac{1}{N} \sum_{i=1}^{N} r_\psi(\tau_i) - \log Z \qquad\qquad Z = \int p(\tau) \exp(r_\psi(\tau)) d\tau$$

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The IRL Partition Function

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estimate with expert samples

The IRL Partiotion Function

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estimate with expert samples

soft optimal policy under current reward

Unknown Dynamics & Large State/Action Spaces

Assume we don't know the dynamics, but we can sample, like in standard RL

recall:

$$\nabla_\psi \mathcal{L} = E_{\tau \sim \pi^\star(\tau)}[\nabla_\psi r_\psi(\tau_i)] - E_{\tau \sim p(\tau|\mathcal{O}_{1:T},\psi)}[\nabla_\psi r_\psi(\tau)]$$

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estimate with expert samples

soft optimal policy under current reward

idea: learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

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estimate with expert samples — soft optimal policy under current reward

idea: learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

$$J(\theta) = \sum_t E_{\pi(\mathbf{s}_t,\mathbf{a}_t)}[r_\psi(\mathbf{s}_t,\mathbf{a}_t)] + E_{\pi(\mathbf{s}_t)}[\mathcal{H}(\pi(\mathbf{a}|\mathbf{s}_t))]$$

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estimate with expert samples

soft optimal policy under current reward

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then run this policy to sample $\{\tau_j\}$

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$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{M}\sum_{j=1}^{M} \nabla_\psi r_\psi(\tau_j)$$

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sum over expert samples — sum over policy samples

More Efficient Sample-Based Updates

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{M}\sum_{j=1}^{M} \nabla_\psi r_\psi(\tau_j)$$

sum over expert samples

sum over policy samples

learn $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm

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looks expensive! what if we use "lazy" policy optimization?

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improve ~~learn~~ $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm
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More Efficient Sample-Based Updates

$$\nabla_\psi \mathcal{L} \approx \underbrace{\frac{1}{N}\sum_{i=1}^{N}}_{\text{sum over expert samples}} \nabla_\psi r_\psi(\tau_i) - \underbrace{\frac{1}{M}\sum_{j=1}^{M}}_{\text{sum over policy samples}} \nabla_\psi r_\psi(\tau_j)$$

improve (a little) ~~learn~~ $p(\mathbf{a}_t|\mathbf{s}_t, \mathcal{O}_{1:T}, \psi)$ using any max-ent RL algorithm
then run this policy to sample $\{\tau_j\}$

looks expensive! what if we use "lazy" policy optimization?

More Efficient Sample-Based Updates

$$\nabla_\psi \mathcal{L} \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\psi r_\psi(\tau_i) - \frac{1}{M} \sum_{j=1}^{M} \nabla_\psi r_\psi(\tau_j)$$

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each policy update w.r.t. r_ψ brings us closer to the target distribution!

Guided Cost Learning Algorithm (Finn et al. ICML ’16)

initial
policy π

human
demonstrations

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initial
policy π

generate policy
samples from π

human
demonstrations

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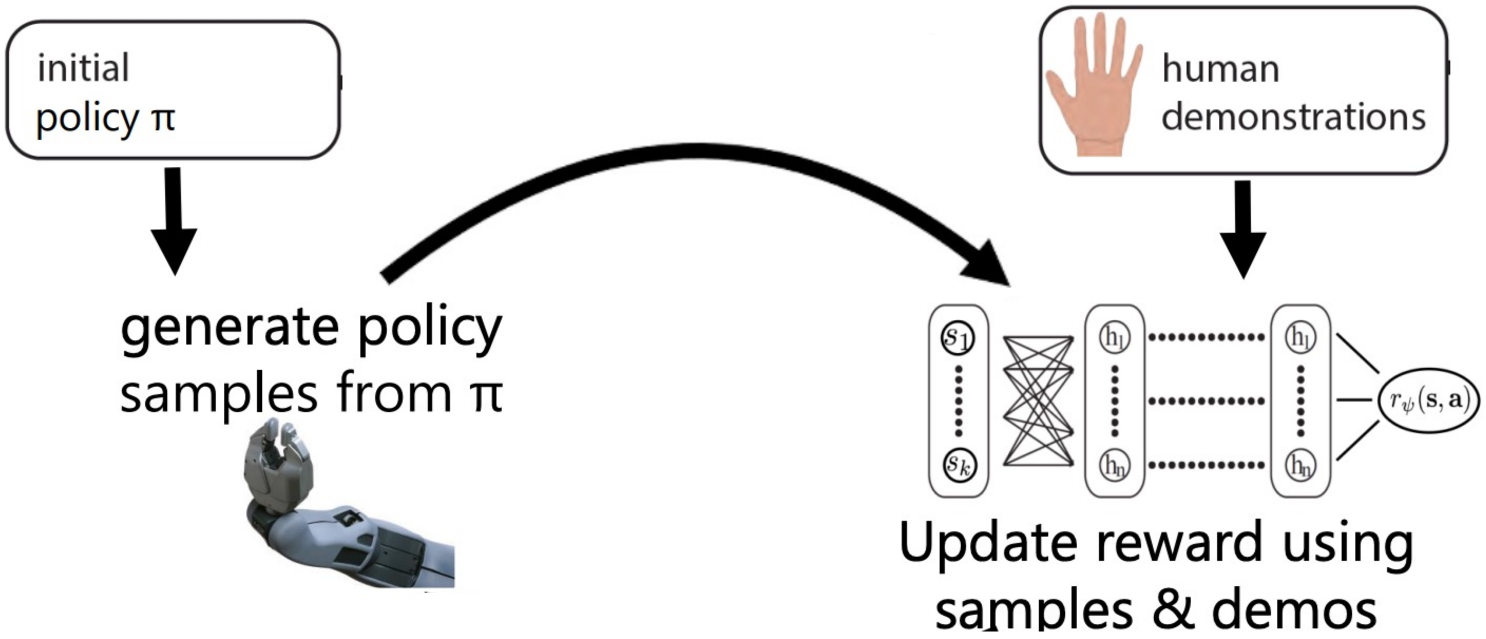
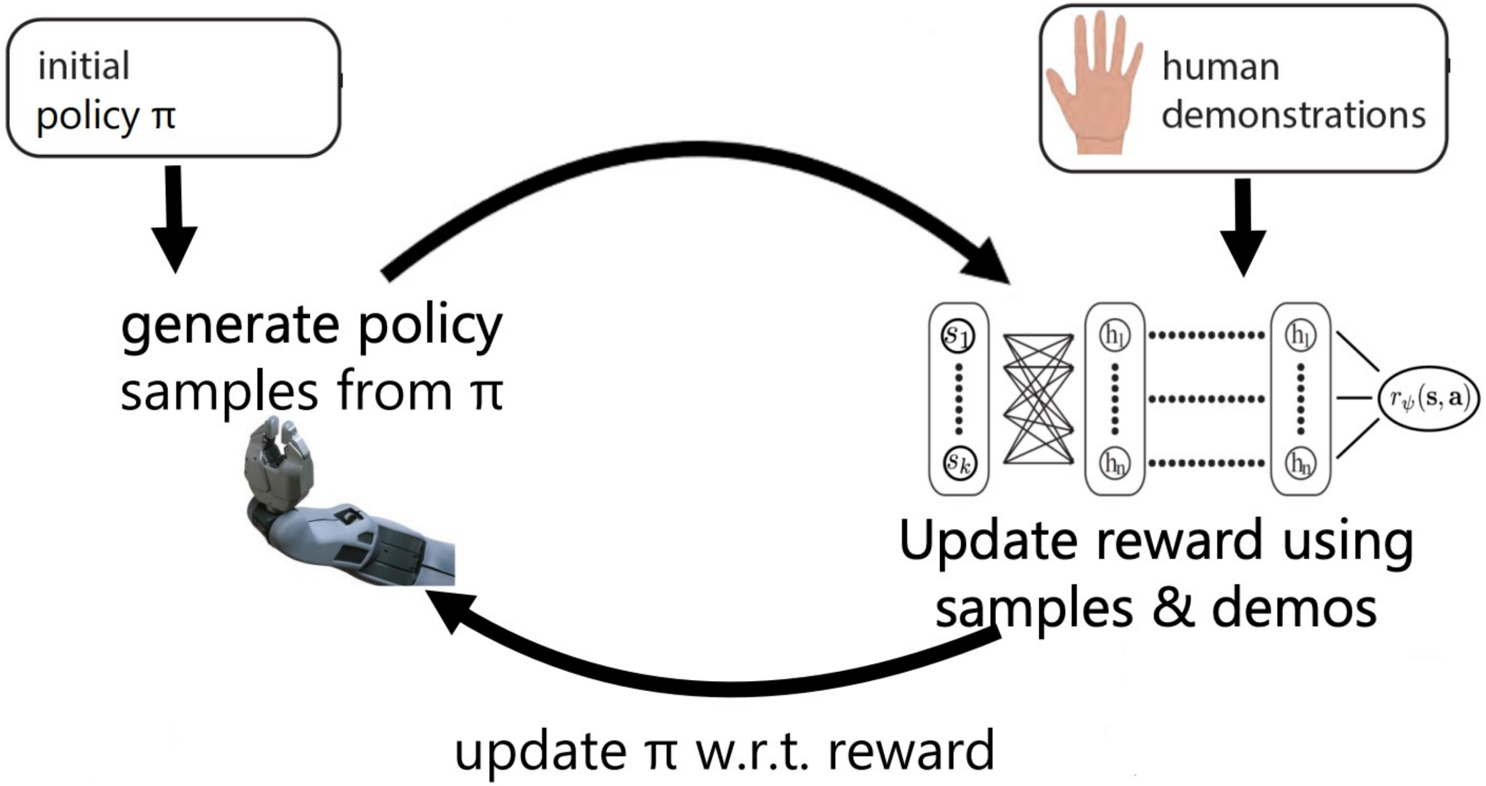


Guided Cost Learning Algorithm (Finn et al. ICML ’16)



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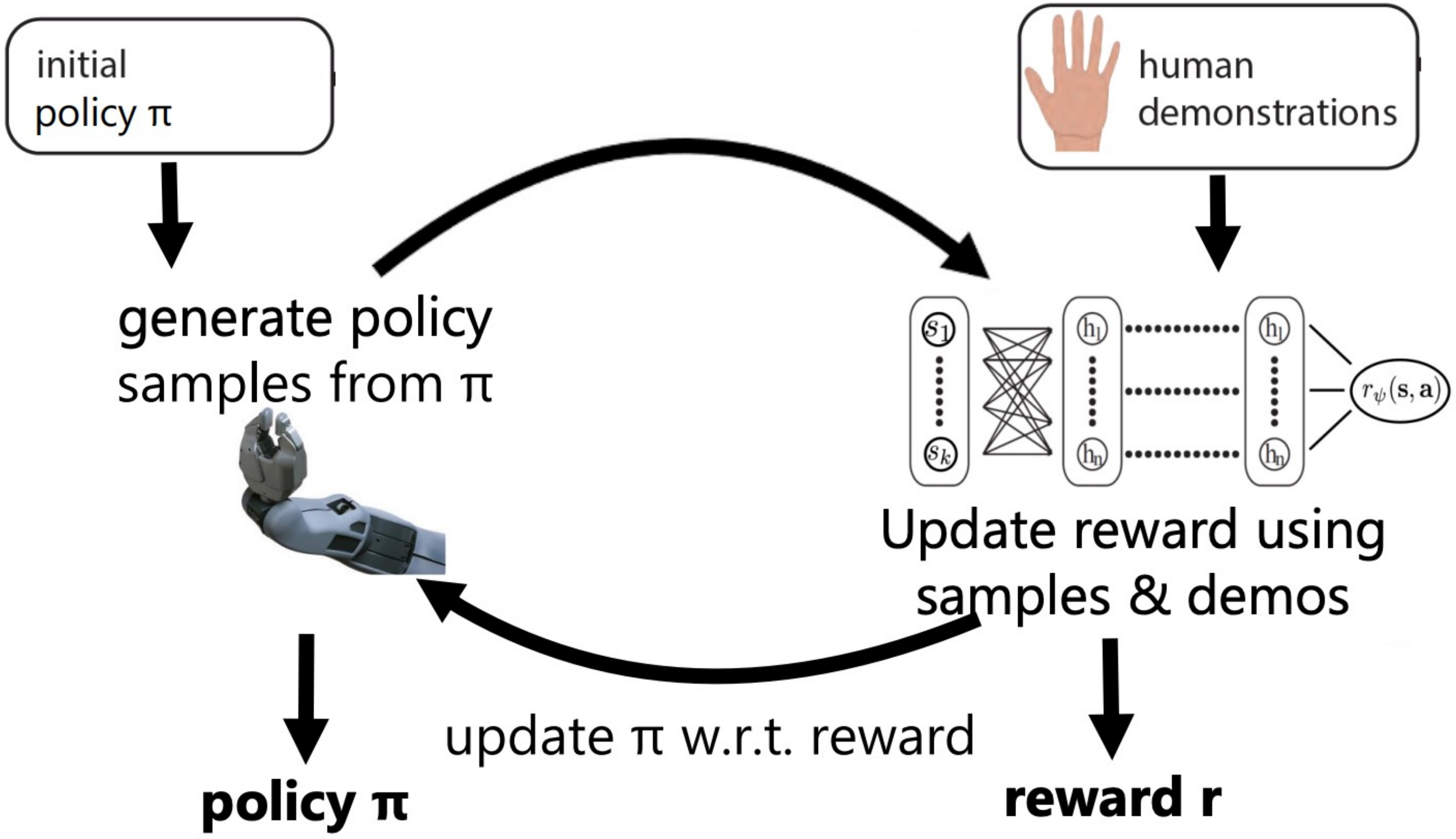

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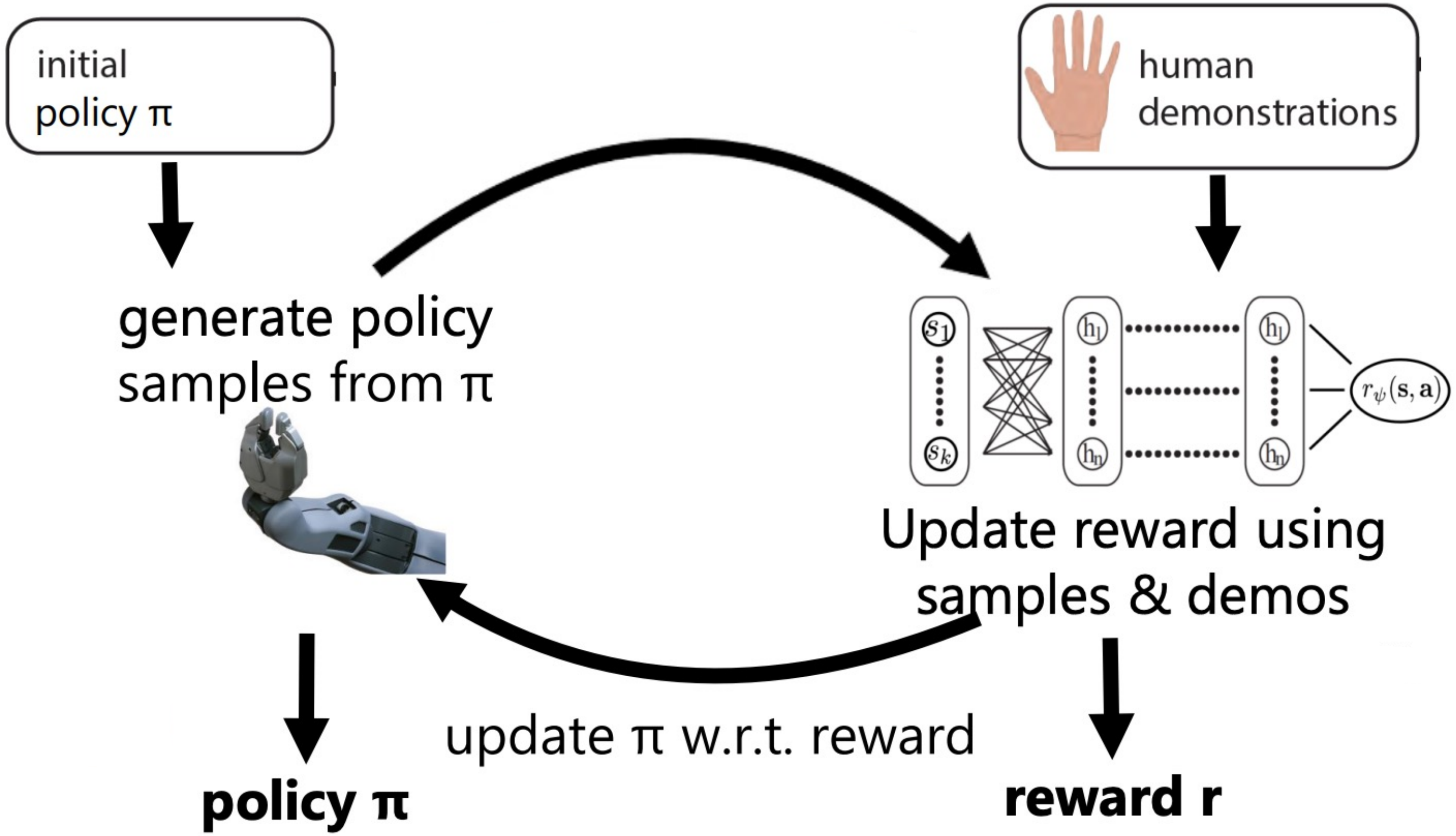

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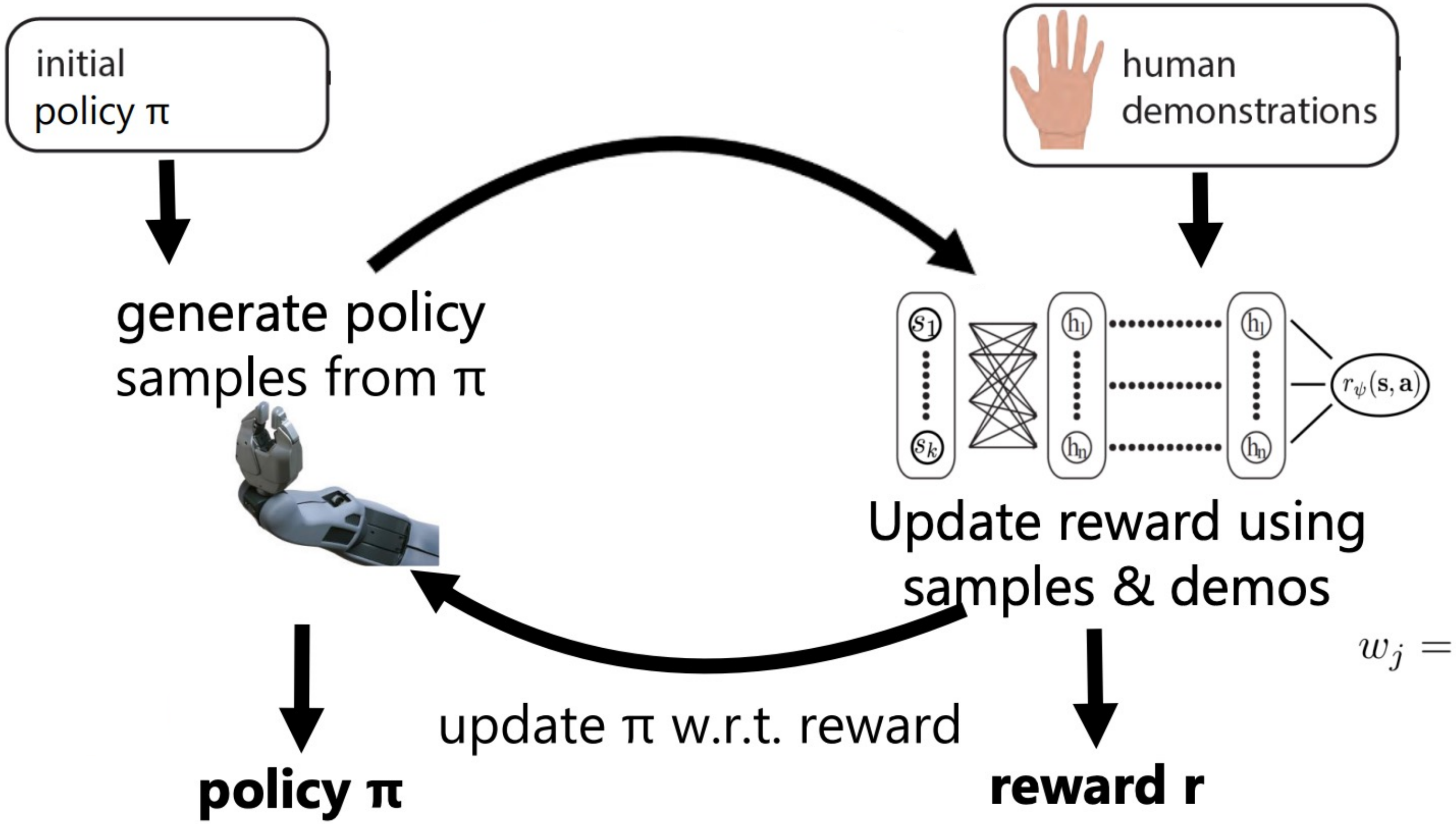

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Suggested Reading on Inverse RL

<u>Classic Papers</u>:
Abbeel & Ng ICML '04. *Apprenticeship Learning via Inverse Reinforcement Learning*. Good introduction to inverse reinforcement learning
Ziebart et al. AAAI '08. *Maximum Entropy Inverse Reinforcement Learning*. Introduction to probabilistic method for inverse reinforcement learning

<u>Modern Papers</u>:
Finn et al. ICML '16. *Guided Cost Learning*. Sampling based method for MaxEnt IRL that handles unknown dynamics and deep reward functions
Wulfmeier et al. arXiv '16. *Deep Maximum Entropy Inverse Reinforcement Learning*. MaxEnt inverse RL using deep reward functions
Ho & Ermon NIPS '16. *Generative Adversarial Imitation Learning*. Inverse RL method using generative adversarial networks
Fu, Luo, Levine ICLR '18. Learning Robust Rewards with Adversarial Inverse Reinforcement Learning