Reinforcement Learning

Computer Engineering Department
Sharif University of Technology

Mohammad Hossein Rohban, Ph.D.

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Courtesy: Some slides are adopted from CS 285 Berkeley, and CS 234 Stanford, and Pieter Abbeel's compact series on RL.

Disadvantages of Monte-Carlo Learning

- We have seen MC algorithms can be used to learn value predictions
- But when episodes are long, learning can be slow
  - we have to **wait until an episode ends** before we can learn…
  - return can have **high variance**
    - Which one is more? First-visit or every-visit
- Are there alternatives? (Spoiler: yes)

Monte Carlo Control

Prediction → V^{π}, Q^{π}

Monte-Carlo Control

Repeat:

Pred.

- Sample episode $1, \ldots, k, \ldots,$ using π: $\{S_1, A_1, R_2, \ldots, S_T\} \sim \pi$
- For each state S_t and action A_t in the episode:

$$q(S_t, A_t) \leftarrow q(S_t, A_t) + \alpha_t \left(G_t - q(S_t, A_t)\right)$$

- e.g.,

$$\alpha_t = \frac{1}{N(S_t, A_t)} \quad \text{of} \quad \alpha_t = 1/k$$

Control

- Improve policy based on new action-value function

$$\pi^{new}(s) = \underset{a \in A}{\operatorname{argmax}}\, q(s, a)$$

Any issue?

- Let's consider this example:
- Discount = 1, start in state H.

x

A	B	C	D	E	F	G	H	I	J
r=10	→	→	→	→	→	↑	→	→	r=1

H, →, 0, I, →, 1, J

$Q(H, \rightarrow) = 1$ $Q(I, \rightarrow) = 1$

$\forall s,a \; Q(s, a) = 0$

$\forall s \quad \pi(s) = \arg\max_a Q(s,a)$

Epsilon Greedy Policy

- Simple idea to balance exploration and achieving rewards
- Let $|A|$ be the number of actions
- Then an ϵ-greedy policy w.r.t a state action value $Q(s, a)$ is

$\pi(a|s) =$

- $\arg\max_a Q(s, a)$, w. prob $1 - \epsilon + \frac{\epsilon}{|A|}$
- $a' \neq \arg\max Q(s, a)$ w. prob $\frac{\epsilon}{|A|}$

Does this hurt improvement?

Theorem

For any ϵ-greedy policy π_i, the ϵ-greedy policy w.r.t. Q^{π_i}, π_{i+1} is a monotonic improvement $V^{\pi_{i+1}} \geq V^{\pi_i}$

Monte-Carlo Control (done right)

Repeat:

- Sample episode $1, \ldots, k, \ldots$, using π: $\{S_1, A_1, R_2, \ldots, S_T\} \sim \pi$
- For each state S_t and action A_t in the episode:

$$q(S_t, A_t) \leftarrow q(S_t, A_t) + \alpha_t \left(G_t - q(S_t, A_t)\right)$$

- e.g.,

$$\alpha_t = \frac{1}{N(S_t, A_t)} \quad \text{of} \quad \alpha_t = 1/k$$

- Improve policy based on new action-value function

$\pi(a|s) =$

- $\arg\max_a Q(s, a)$, w. prob $1 - \epsilon + \frac{\epsilon}{|A|}$
- $a' \neq \arg\max Q(s, a)$ w. prob $\frac{\epsilon}{|A|}$

Disadvantages of MC Learning

- We have seen MC algorithms can be used to learn value predictions
- But when episodes are long, learning can be slow
  - ...we have to **wait until an episode ends** before we can learn
  - ...return can have **high variance**
- Are there alternatives? (Yes)

Temporal Difference Learning

Prediction

TD Overview

TD methods learn directly from episodes of experience

TD is *model-free*: no knowledge of MDP transitions / rewards

TD learns from *incomplete* episodes, by *bootstrapping*

TD updates a guess towards a guess

$$\hat{V}^{\pi}(s) = \frac{1}{n} \sum_{i=1}^{n} \left[R(s, \pi(s), s_i') + \gamma \hat{V}^{\pi}(s_i') \right]$$

Temporal Difference Learning by Sampling Bellman Equations

- Bellman update equations:

$$v_{k+1}(s) = \mathbb{E}\left[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s, A_t \sim \pi(S_t)\right]$$

- We can sample this!

$$v_{t+1}(S_t) = R_{t+1} + \gamma v_t(S_{t+1})$$

- Samples could be averaged, in a similar way to MC:

$$v_{t+1}(S_t) = v_t(S_t) + \alpha_t \underbrace{\Big(\underbrace{R_{t+1} + \gamma v_t(S_{t+1})}_{\text{target}} - v_t(S_t) \Big)}$$

temporal difference error δ_t

Temporal Difference Learning

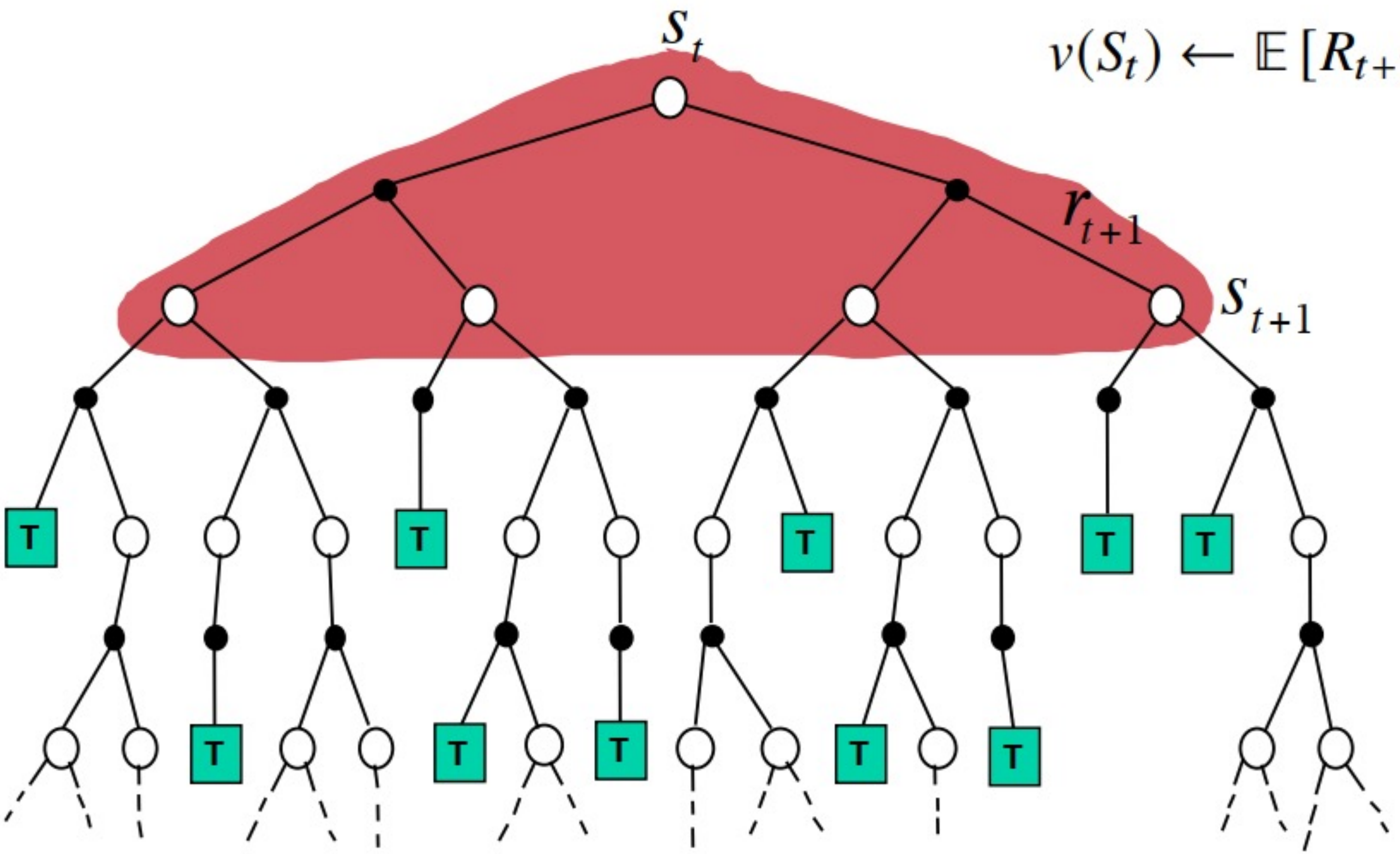
- Prediction setting: learn v_π online from experience under policy π
- Monte Carlo
  - Update value $v_n(S_t)$ towards sampled return G_t

$$v_{n+1}(S_t) = v_n(S_t) + \alpha\left(G_t - v_n(S_t)\right)$$

- TD Learning
  - Update value $v_t(S_t)$ towards estimated return $R_{t+1} + \gamma v(S_{t+1})$

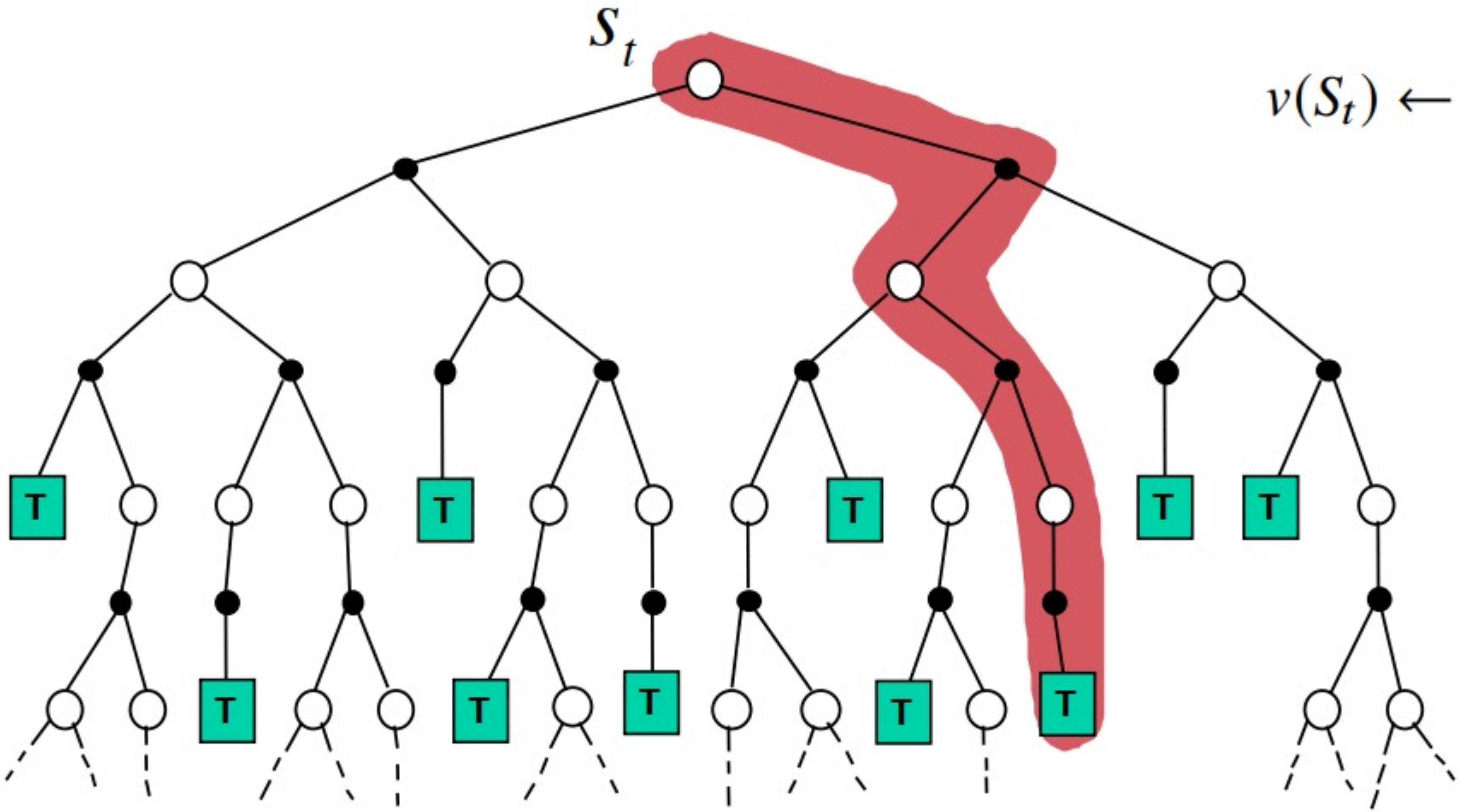
$$v_{t+1}(S_t) \leftarrow v_t(S_t) + \alpha\left(\overbrace{\underbrace{R_{t+1} + \gamma v_t(S_{t+1})}_{\text{target}} - v_t(S_t)}^{\text{TD error}}\right)$$

Backup (Dynamic Programming)



$$v(S_t) \leftarrow \mathbb{E}\left[R_{t+1} + \gamma v(S_{t+1}) \mid A_t \sim \pi(S_t)\right]$$

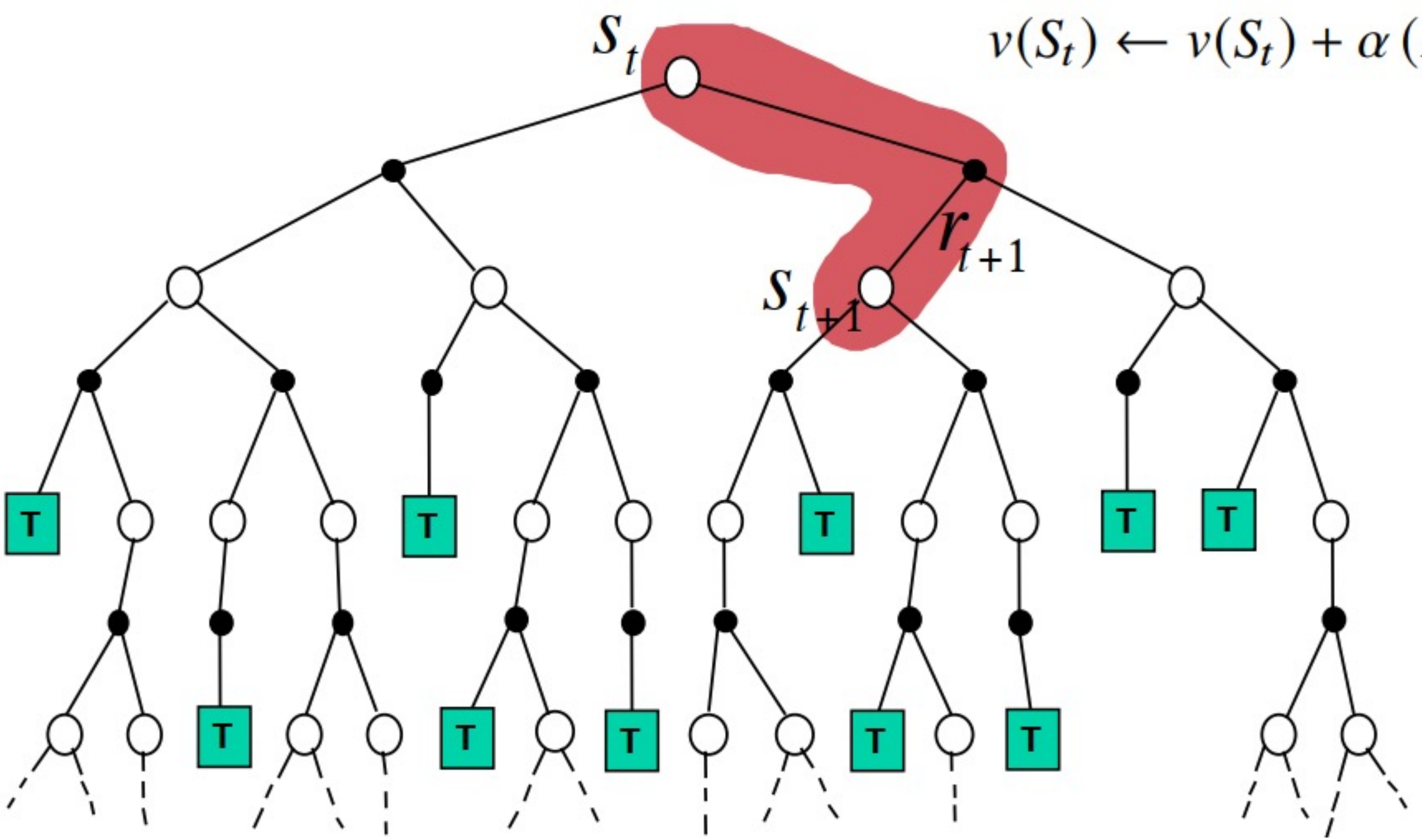
Backup (Monte Carlo)



$$v(S_t) \leftarrow v(S_t) + \alpha\,(G_t - v(S_t))$$

Backup (Temporal Difference)

$$v(S_t) \leftarrow v(S_t) + \alpha\left(R_{t+1} + \gamma v(S_{t+1}) - v(S_t)\right)$$



Bootstrapping and Sampling

- Bootstrapping: update involves an estimate
  - MC does not bootstrap
  - DP bootstraps
  - TD bootstraps
- Sampling: update samples an expectation
  - MC samples
  - DP does not sample
  - TD samples

TD Learning for action values

- We can apply the same idea to action values
- Temporal-difference learning for action values:
  - Update value $q_t(S_t, A_t)$ towards estimated return $R_{t+1} + \gamma q(S_{t+1}, A_{t+1})$

$$q_{t+1}(S_t, A_t) \leftarrow q_t(S_t, A_t) + \alpha \left(\overbrace{\underbrace{R_{t+1} + \gamma q_t(S_{t+1}, A_{t+1})}_{\text{target}} - q_t(S_t, A_t)}^{\text{TD error}} \right)$$

TD vs. MC

- **TD can learn before knowing the final outcome**
  - TD can learn online after every step
  - MC must wait until end of episode before return is known
- **TD can learn without the final outcome**
  - MC must wait until end of episode before return is known
  - MC can only learn from complete sequences
  - TD works in continuing (non-terminating) environments
  - MC only works for episodic (terminating) environments
- **TD is independent of the temporal span of the prediction**
  - TD can learn from single transitions
  - MC must store all predictions (or states) to update at the end of an episode
- **TD needs reasonable value estimates**

Temporal Difference Learning

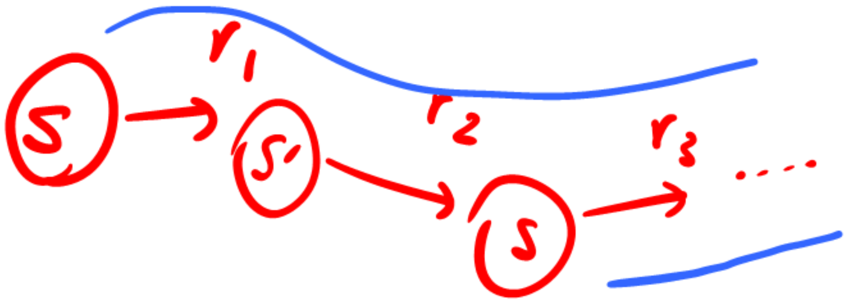
Control

SARSA Algorithm for On-Policy Control

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

$V(s)$

$V(s')$



Off-Policy TD and Q-Learning

On and Off-Policy Learning

- On-policy learning
    - "Learn on the job“
    - Learn about policy π from experience sampled from π
- Off-policy learning
    - "Look over someone’s shoulder“
    - Learn about policy π from experience sampled from μ

Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s, a)$
- While using behavior policy $\mu(a, s)$ to generate actions
- Why is this important?
  - Learn from observing humans or other agents (e.g., from logged data)
  - Re-use experience from old policies (e.g., from your own past experience)
  - Learn about multiple policies while following one policy
  - Learn about greedy policy while following exploratory policy

Q-Learning

- Q-learning estimates the value of the greedy policy

$$q_{t+1}(s,a) = q_t(S_t, A_t) + \alpha_t \left(R_{t+1} + \gamma \max_{a'} q_t(S_{t+1}, a') - q_t(S_t, A_t) \right)$$

- Acting greedy all the time would not explore sufficiently

Theorem

Q-learning control converges to the optimal action-value function, $q \rightarrow q^*$, as long as we take each action in each state infinitely often.

- Note: no need for greedy behavior!
- Works for **any** policy that eventually selects all actions sufficiently often

Q-Learning for Off-Policy Control

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$;
  until S is terminal

(S, A, S', R)

random Prob ϵ

$\arg\max_a Q(s,a)$ $1-\epsilon$

$\pi(s) = \arg\max_{a'} Q(s,a')$