



Policy Gradient Methods

- Policy gradient methods aim to learn the policy directly without using a value function.
- The objective function is the expected sum of rewards from an initial state, denoted as $J(\theta) = E[\sum R_t]$, where θ represents the policy parameters.
- The policy is often parameterized using a neural network, where the inputs are states, and the outputs are probability distributions over actions.
- The goal is to optimize $J(\theta)$ by adjusting θ using gradient ascent.
- Monte Carlo methods are used to approximate the expectation in the objective function.
- **Log trick:** A mathematical transformation is applied to compute the policy gradient:

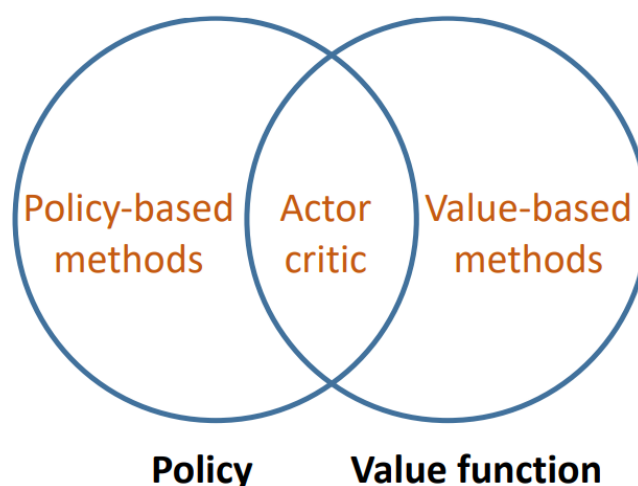
$$\nabla_{\theta} J(\theta) = E \left[\sum \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) R_t \right] \quad (1)$$

where the log-derivative trick simplifies the gradient computation.

- The **REINFORCE algorithm** follows this approach by sampling trajectories, computing rewards, and updating the policy parameters using stochastic gradient ascent.

Issues with Policy Gradient Methods

- **High variance:** The policy gradient estimate has high variance, making convergence slow and unstable.
- **Sample inefficiency:** Since each update discards old samples, a large number of episodes is needed to achieve good performance.
- **Imitation Learning vs. Policy Gradient:**
 - **Imitation learning (Behavioral Cloning)** trains a policy to mimic expert demonstrations using supervised learning.
 - **Policy Gradient** optimizes policy based on rewards rather than imitation.
 - A major difference is that policy gradients involve weighting by reward, while imitation learning lacks such weighting.



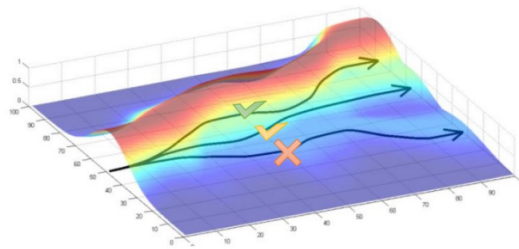


Model-Free RL Overview

- **Value-based methods:** Learn value functions (e.g., Q-learning, Deep Q-Networks) but do not learn policies directly.
- **Policy-based methods:** Learn policies directly without using value functions.
- **Actor-Critic methods:** Combine value-based and policy-based methods by learning both a policy (actor) and a value function (critic).

Policy Gradient Intuition

- The fundamental idea is simple:
 - Actions that lead to higher rewards should become more likely.
 - Actions that lead to lower rewards should become less likely.
 - This formalizes the concept of "trial and error" learning.



Bias and Variance in Policy Gradient

- Policy gradient estimation is **unbiased**, but it suffers from **high variance**, making optimization inefficient.
- The main sources of high variance are:
 - Monte Carlo return estimation.
 - Long-term dependencies on past actions and states.

Reducing Variance in Policy Gradient Estimation

Several techniques can be used to reduce variance while preserving unbiasedness:

1. Causality Trick ("Reward-to-Go")

- Instead of using the total return from the beginning of the trajectory, we only use future rewards:

$$\nabla_{\theta} J(\theta) = E \left[\sum_{l=0}^T \nabla_{\theta} \log \pi_{\theta}(a_l | s_l) \sum_{t'=l}^T R_{t'} \right] \quad (2)$$

- This reduces variance because an action taken at time t is only responsible for rewards obtained after t , not before.
- Introduces a discount factor γ (typically around 0.99) to reduce the impact of distant rewards:

$$\nabla_{\theta} J(\theta) = E \left[\sum_{l=0}^T \nabla_{\theta} \log \pi_{\theta}(a_l | s_l) \sum_{t'=l}^T \gamma^{t'-l} R_{t'} \right] \quad (3)$$



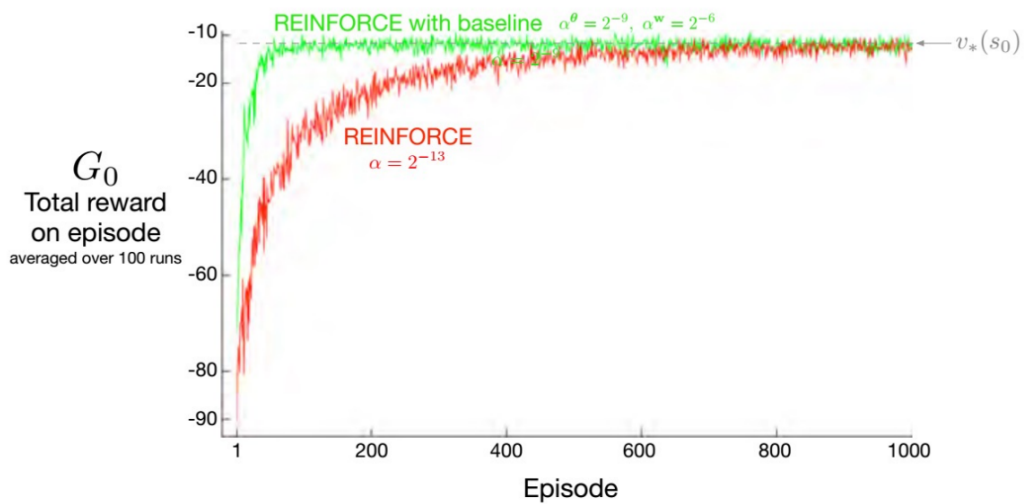
2. Baseline Subtraction

- The gradient formula can be modified by subtracting a baseline $b(s_t)$:

$$\nabla_{\theta} J(\theta) = E \left[\sum_{t=0}^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) (R_t - b(s_t)) \right] \quad (4)$$

- If $b(s_t)$ is chosen properly, it does not introduce bias but can reduce variance significantly.
- A common choice is the **value function** $V(s_t)$, leading to the **Advantage Function**:

$$A(s_t, a_t) = Q(s_t, a_t) - V(s_t) \quad (5)$$



4. Actor-Critic Methods

- Actor** updates the policy based on the advantage function.
- Critic** estimates the value function to compute the advantage.
- Advantage Estimation:**

$$A(s_t, a_t) = R_t - V(s_t) \quad (6)$$

- This method balances bias and variance better than pure policy gradient or value-based methods.